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ORIGINAL ARTICLE

# Planning paths to multiple targets: memory involvement and planning heuristics in spatial problem solving

J. M. Wiener · N. N. Ehbauer · H. A. Mallot

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**Abstract** For large numbers of targets, path planning is a complex and computationally expensive task. Humans, however, usually solve such tasks quickly and efficiently. We present experiments studying human path planning performance and the cognitive processes and heuristics involved. Twenty-five places were arranged on a regular grid in a large room. Participants were repeatedly asked to solve traveling salesman problems (TSP), i.e., to find the shortest closed loop connecting a start location with multiple target locations. In Experiment 1, we tested whether humans employed the nearest neighbor (NN) strategy when solving the TSP. Results showed that subjects outperform the NN-strategy, suggesting that it is not sufficient to explain human route planning behavior. As a second possible strategy we tested a hierarchical planning heuristic in Experiment 2, demonstrating that participants first plan a coarse route on the region level that is refined during navigation. To test for the relevance of spatial working memory (SWM) and spatial long-term memory (LTM) for planning performance and the planning heuristics applied, we varied the memory demands between conditions in Experiment 2. In one condition the target locations were directly marked, such that no memory was required; a second condition required participants to memorize the target locations during path planning (SWM); in a third condition, additionally, the locations of targets had to retrieved from LTM (SWM and LTM). Results showed that navigation performance decreased with increasing memory demands while the dependence on the hierarchical planning heuristic increased.

## Introduction

Planning short paths to multiple targets can be complex and computationally expensive. This is best demonstrated by the traveling salesman problem (TSP) that can be stated as follows: given a number of locations and the costs (here distance) of traveling between them, what is the cheapest round trip route that visits each location once. The number of possible round trips is computed as $(N - 1)!/2$, with $N$ being the number of locations. For six locations, 60 different round trips are possible, for ten locations, already 181,440 round trips exist. The TSP belongs to the class of NP-hard problems for which no algorithm exists for calculating the optimal solution within a practical time if $N$ is large (Lawler, Lenstra, Kan, & Schmoys, 1985). For humans, TSP-like planning tasks are actually quite common in our day-to-day wayfinding, for example, on a typical shopping route on which several shops are visited (e.g., Gärling & Gärling, 1988). Obviously, rather than actually calculating and comparing all possible solutions, we rely on simplifying processes that reduce cognitive effort while resulting in reasonably short solutions (c.f., Dry, Lee, Vickers, & Hughes, 2006). Such simplifying processes that replace complex or computationally expensive algorithms have been termed *heuristics* (Newell & Simon, 1972), and have primarily been studied in the context of judgment and decision making (e.g., Gigerenzer, Todd, & the ABC Research Group, 1999). Shah and Oppenheimer (2008) recently suggested that heuristics research should focus on investigating *how* people reduce the effort

J. M. Wiener (✉) · N. N. Ehbauer · H. A. Mallot
Cognitive Neuroscience, University of Tübingen,
Auf der Morgenstelle 28, 72076 Tübingen, Germany
e-mail: mail@jan-wiener.net

J. M. Wiener
Center for Cognitive Science, University of Freiburg,
Friedrichstrasse 50, 79098 Freiburg, Germany

associated with complex decision processes. In this work we investigated planning heuristics that humans apply when solving complex TSP-like tasks that require movements through space, and we describe a hierarchical planning strategy that reduces cognitive effort on several levels. Additionally, as path planning for actual navigation usually requires spatial working memory (SWM) as well as spatial long term memory (LTM), we systematically varied memory demands to investigate the impact of the different memory systems for planning performance and planning processes as well as interrelations between the memory systems and planning heuristics.

Spatial optimization in the TSP has primarily been investigated by means of visual versions, in which participants are presented with a number of identical dots on a computer screen. Their task is to connect the dots such that the resulting path is optimal with respect to overall length (e.g., MacGregor & Ormerod, 1996; MacGregor, Ormerod, & Chronicle, 1999; Van Rooij, Stege, & Schactman, 2003; Graham, Joshi, & Pizlo, 2000; Vickers, Lee, Dry, & Hughes, 2003b; Vickers, Lee, Dry, Hughes, & McMahon, 2006; Pizlo et al., 2006; Gibson, Wasserman, & Kamil, 2007; Kong & Schunn, 2007). Generally, results from these studies show that humans reach very good performance levels, often with near optimal solutions (e.g., Vickers, Butavicius, Lee, & Medvedev, 2001). There is an ongoing debate on the optimization-strategies applied in these experiments: MacGregor and Ormerod (1996) have proposed that participants used the convex hull as part of their strategy (see also MacGregor, Ormerod, & Chronicle, 2000, 2004). They argue that the fact that a tour that follows the convex hull method is free of crossings and that humans tend to avoid crossings is one important piece of supporting evidence for this method. Van Rooij et al. (2003), however, argue that participants know that crossings will result in sub-optimal solutions. Hence, they avoid crossings when solving TSPs, rather than following the convex hull method. Vickers, Bovet, Lee, and Hughes, (2003a) proposed a hierarchical nearest neighbor (NN) method, assuming that participants first establish clusters of several dots based on NN distances, which they then sequentially link into a tour, using some variant of the NN algorithm. Graham et al. (2000) proposed another hierarchical model, assuming that from the original stimulus (dot pattern) a series of images are generated which are increasingly blurred and compressed. By these means a hierarchy of images is generated in which neighboring points collapse to clusters. The algorithm then starts with generating a tour in an image with only three blurred clusters. By progressively moving to the next lower layer in the hierarchy further clusters are inserted into the tour, eventually reaching the level of single dots. A common feature shared by many of the approaches presented above is that the spontaneous perceptual organization when perceiving the stimulus pattern can be assumed to play a critical role in solving visual TSPs (see Dry et al., 2006).

Visual TSPs are conducted in figural or pictorial space (Montello, 1993; Hegarty, Montello, Richardson, Ishikawa, & Lovelace, 2006) in which spatial relations between participants and relevant locations remain constant. Usually only relevant locations are presented, all information required to solve the task is visually accessible, and the chosen paths are displayed while solving the TSP. Hence, no memory is required. Path planning during actual navigation, however, has very different characteristics (for a comparison of path planning at different scale levels, see Wiener & Tenbrink, 2008): most importantly, it takes place in larger spaces, i.e., vista or environmental spaces (c.f. Montello, 1993), resulting in a variety of cognitive processes and memory demands that are absent in figural space. Due to movements, for example, spatial relations between navigator and the surrounding constantly change and relevant locations get out of sight. This requires memorizing and updating their positions during navigation. Furthermore, given that no external representation of space is available, the actual locations of relevant places in large scale, environmental spaces (such as cities) has to be retrieved from spatial LTM rather than from perception. Additionally, in order to plan paths covering multiple locations, these locations have to be simultaneously activated and represented in a SWM during the actual planning process. Only few studies investigated path planning and optimization with multiple target locations in the context of actual navigation. And, to the authors' knowledge, no study so far systematically investigated the role and impact of the various memory related constraints on path planning performance, planning processes, and the planning heuristics applied.

Gärling and Gärling (1988) demonstrated that most shoppers who minimized the total distance of their shopping routes employed strategies similar to the NN algorithm (see also Gärling, Säisä, Böök, & Lindberg, 1986), a simple algorithm for solving TSP-like tasks quickly: from its current location, the NN algorithm repeatedly visits the closest target that has not been visited before until all target locations have been visited (e.g., Golden, Bodin, Doyle, & Stewart, 1980). Wiener and Mallot (2003) demonstrated that environmental regions influenced navigation behavior when planning short paths to visit multiple targets: participants minimized the number of region boundaries they crossed during navigation and preferred paths that allowed for fastest access to the region containing the target. These results corroborate findings suggesting that regional information is explicitly represented in spatial memory (cf. Stevens & Coupe, 1978; Hirtle & Jonides, 1985; McNamara, 1986), and show that such information is taken into account during planning. In everyday navigation, multiple information sources are available that allow for various planning strategies. In a series of navigation experiments, Wiener, Schnee, and Mallot (2004) studied the use and interaction of different planning strategies. In addition to the *region-based* planning strategy sketched above, two further strategies, the *cluster*-strategy and the *least-decision-load* strategy, were identified. The cluster-strategy states that neighboring places are grouped together to form clusters. If two clusters are equidistant, routes are planned such that the larger cluster is visited first, by this means increasing the number of visited targets as fast as possible (c.f. Gallistel & Cramer, 1996). The least-decision-load strategy states that the number of possible movement decisions along a path is taken into account during planning. This strategy predicts preferences for paths that minimize possible movement decisions along the path. It could be employed, because the risk of getting lost is smaller on less complex routes. The NN-, the cluster-, and the region-based-strategy are heuristics that reduce mental effort during planning, either by planning locally rather than globally (NN-strategy), or by aggregating targets in a first planning step (cluster and region-based approaches), thus reducing search or problem space. At the same time, applying these planning strategies reduces working memory demands during planning and might be a response to memory related constraints that do not play a role in visual TSPs.

## Motivation and synopsis

The objective of this study was to develop an increased understanding of the cognitive components, processes, and heuristics involved in spatial problem solving in a navigational context. As discussed above, path planning in everyday wayfinding usually requires both, spatial LTM and SWM: the positions of target location beyond the current sensory horizon have to be retrieved from spatial LTM; and, if multiple target locations are to be visited, these locations have to be simultaneously activated and maintained in a SWM during the actual planning process. According to the number of targets to visit, working memory related constraints will influence path planning. To gain first insight into the impact of these memory systems on planning performance, planning processes, and planning strategies applied, we asked participants to solve navigational TSPs of different sizes in a large experimental room. Spaces of this scale are referred to as vista spaces (Montello, 1993) as they can be apprehended from a single place. Vista spaces combine features of figural (pictorial) spaces with characteristics of environmental spaces that are crucial for navigation: while the entire environment can be overlooked (as in figural spaces), spatial relations between observer and targets change during locomotion which requires memory and updating processes (as in environmental spaces). Carrying out the experiments in a vista space also allowed us to control the visual accessibility of symbols defining the target locations. By this means we systematically varied the memory demands required (no memory, SWM, and LTM). To solve the TSPs efficiently, participants needed to judge the local distances between any two target locations. It is well-known that spatial memory of large scale (environmental) spaces is subject to systematic distortions (e.g., McNamara & Diwadkar, 1997) and it has been questioned whether humans do possess Euclidean metric spatial knowledge of such spaces (e.g., Foo, Warren, Duchon, & Tarr, 2005; Foo, Duchon, Warren, & Tarr, 2007). There is, however, convincing evidence that humans can quite accurately judge and estimate distances in vista spaces, as used in this study. If they are, for example, shown a target location and are then asked to walk towards it with their eyes closed, they usually end up near the physical target locations (Philbeck, Loomis, & Beall, 1997; Loomis, Klatzky, Philbeck, & Golledge, 1998). Furthermore, in vista space, participants can reliably distinguish paths composed of multiple segments if these paths differed as little as 1.7% in total length (Wiener, Lafon, & Berthoz, 2008). Knowledge about local distances between goal locations is sufficient for solving the TSPs in this study; full metric embeddings as are discussed by Foo et al. (2005) are not required.

Experiment 1 pursued two main purposes. First, it was designed to establish the novel approach and to test for participants' general performance in solving navigational TSPs in vista space. For this, participants' performance of finding the shortest path in TSPs with varying number of targets was evaluated. Second, the experiment examined two simple planning strategies, the NN strategy and the cluster-strategy (see above) that have been suggested to be involved in visual TSPs as well as in path planning in large scale spaces. Interviews with participants after the experiments allowed for insights into further planning strategies particularly relevant in the current experimental approach. This information was used to design Experiment 2, in which memory demands were systematically varied between experimental conditions: similar to the visual TSPs, one condition required no memory; a second condition required to memorize the locations of the targets (SWM) during planning; a third condition additionally required to retrieve the target locations from spatial LTM (SWM + LTM). Comparisons of planning performance and the usage of planning strategies and heuristics between conditions allowed investigating the influence of the different memory systems on spatial problem solving.

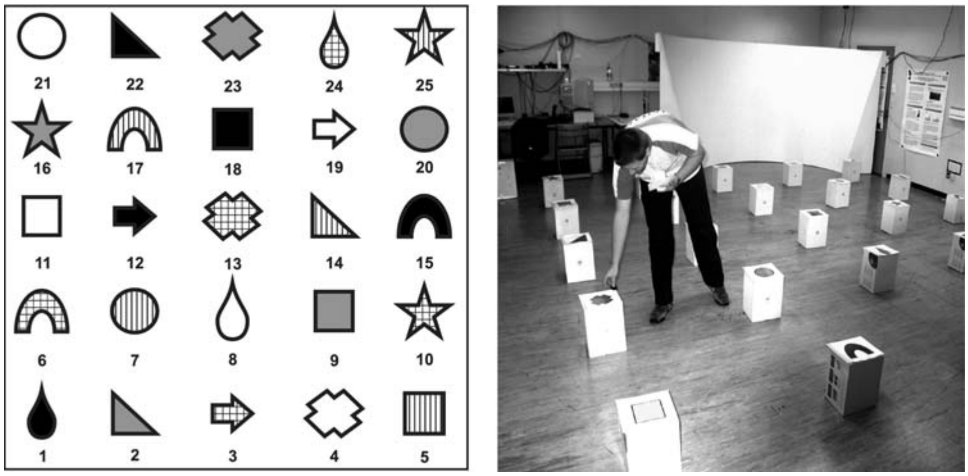


**Fig. 1** *Left* schematic drawing of the experimental setup. The image is adapted to represent the original colors of the symbols (black, yellow, red, green, blue) in *black-and-white*, *right* participant solving a navigation task

## Experiment 1

### Materials and methods

#### *The experimental setup*

The experiment was conducted in a 6.0 × 8.4 m experimental room. Twenty-five small cardboard boxes were arranged on a 5 × 5 squared grid with a mesh size of 1.1m. Twenty-five symbols were randomly distributed about the 25 pillars (see Fig. 1). In order to control for effects of the specific symbol-configuration, two versions of the setup were created that differed only in the specific arrangement of the symbols. Half of the participants conducted the experiment in one configuration, the other half conducted the experiment in the alternative configuration.

#### *Procedure*

Participants were repeatedly asked to solve TSPs. For each TSP they received a '*shopping list'* depicting the symbol defining the start location and the symbols defining the target locations (Fig. 2). Participants were given the lists in random order, one at a time, and upside-down. They were verbally informed about the start location and asked to move to that location. Only after reaching the start location, they were allowed to turn around the *shopping list* and the trial started. The participants' task was to navigate the shortest route connecting the start location with all target locations and return to the start location assuming straight line distances between target locations. During navigation, they kept the shopping list and marked visited target locations by placing little black markers on the cardboard pillars.

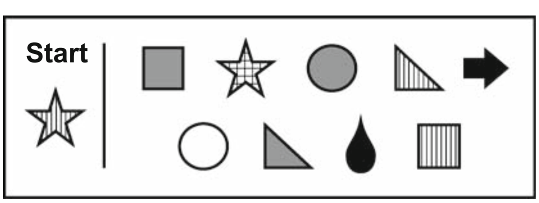


**Fig. 2** Example of a *shopping list* for a navigation task with start place and nine target locations (equivalent to a TSP size of 10)

To control for the influence of the specific order of the symbols depicted on the *shopping list*, two versions of each shopping list were generated. Half of the participants received one version of the shopping lists, the other half received the other version.

#### *Types of navigation tasks*

Each participant solved 36 different TSPs consisting of a start location plus 4, 5, 6, 7, 8 or 9 target places (TSP sizes therefore range between 5 and 10) in random order. The TSPs could be subdivided into three types, *NN-adequate tasks*, *NN-inadequate tasks*, and *cluster-tasks (NN-ambiguous tasks)* (see Fig. 3; Table 1 for a complete list of all TSPs).

1. *NN-adequate/inadequate tasks* For NN-adequate tasks, the predictions of the NN algorithm were identical with the optimal (i.e., the shortest possible) path. For NN-inadequate tasks, the NN algorithm did predict a clearly sub-optimal path (see Fig. 3).
2. *Cluster tasks (NN-ambiguous)* Here target locations were distributed in two distinct clusters of unequal size. These TSPs were NN-ambiguous (i.e., the NN-algorithm did not make clear predictions for a single path): the closest target locations were always equidistant from the starting place and similar situations reoccurred during navigation (i.e., close target locations were equidistant from the current position).

#### *Participants*

Twenty-four participants (12 women, mean age: 22.88 years) participated in the experiment. They were mostly university students and were paid 8 Euro an hour.

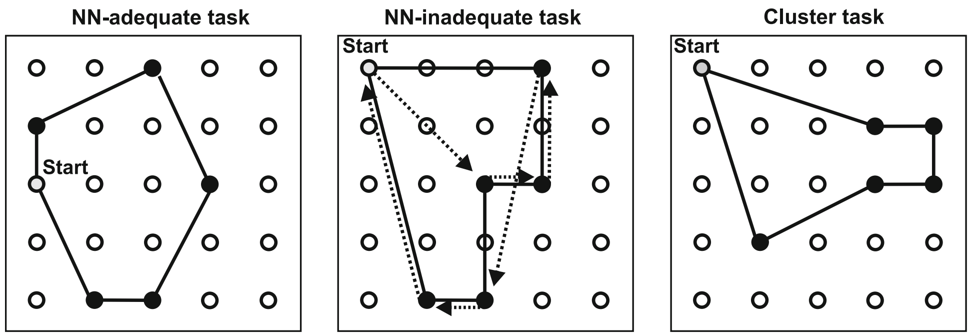


**Fig. 3** Example TSPs with start place and five target locations (TSP size is 6) for the *NN-adequate tasks* (*left*), the *NN-inadequate tasks* (*middle*), and the *cluster tasks* (*right*). Start locations are represented by *grey circles*, target locations are represented by *solid black circles*, and *black lines* depict the optimal paths. For the NN-inadequate task the prediction of the NN-strategy is displayed by the *dashed lines*

**Table 1** The table lists all navigation tasks of Experiment 1

| Navigation tasks | Number of target places | Start place (target places) |
|---|---|---|
| NN-ambiguous (cluster) | 4 | 3 (17,16,22,19), 15 (8,17,23,18) |
| | 5 | 21 (19,20,15,14,7), 6 (21,22,10,9,4) |
| | 6 | 4 (1,6,18,23,24,19), 25 (10,5,4,9,8,22) |
| | 7 | 11 (18,24,10,9,4,3,8), 5 (2,1,6,7,19,24,20) |
| | 8 | 23 (14,15,10,5,4,9,6,12), 24 (22,21,16,11,17,14,9,15) |
| | 9 | 20 (9,10,5,4,3,8,17,18,23), 21 (6,1,2,7,8,3,4,20,24) |
| NN-adequate | 4 | 3 (11,22,19,10), 6 (17,24,15,4) |
| | 5 | 11 (16,23,14,3,2), 21 (24,15,4,1,12) |
| | 6 | 22 (23,19,15,8,2,16), 25 (14,9,8,2,6,16) |
| | 7 | 5 (15,25,18,16,11,1,2), 24 (15,10,3,7,11,17,18) |
| | 8 | 20 (19,24,22,12,6,2,4,9), 3 (8,12,11,22,23,19,20,15) |
| | 9 | 25 (20,14,9,10,5,2,1,12,21), 23 (24,25,20,9,7,6,11,16,17) |
| NN-inadequate | 4 | 3 (16,23,25,13), 1 (13,10,19,21) |
| | 5 | 21 (24,14,13,3,2), 11 (21,13,10,4,7) |
| | 6 | 23 (19,10,5,13,7,16), 4 (8,1,11,12,24,20) |
| | 7 | 16 (12,13,19,25,9,8,1), 5 (9,20,13,18,22,16,2) |
| | 8 | 1 (11,22,25,18,13,8,5,2), 3 (2,12,21,18,20,14,9,5) |
| | 9 | 24 (18,21,12,7,6,1,3,5,19,24), 23 (19,14,10,5,8,2,6,16,22,23) |

The start place is followed by the target locations (in brackets). The numbers correspond to the place numbers in the schematized drawing of the experimental environment (see Fig. 1)

*Analysis*

The sequence of places visited were recorded for each TSP and the length of the resulting tour was calculated, assuming linear route segments between target points. For each TSP we also computed the optimal solution by comparing the length of all possible permutations. Performance of planning and executing a short route was assessed by comparing the length of the chosen path with the length of the optimal solution and was described in percentage above optimal (PAO; Wiener et al., 2008). A PAO value of 10 corresponded to a path that was 10% longer than the optimal solution. Furthermore, the percentage of trials in which participants found the optimal solution was calculated (*found shortest route*). For each trial the *start time* (the time from revealing the shopping list until initiating locomotion) was recorded.

Due to the large number of main effects and interactions tested in this experiment, an $\alpha$ level of 0.01 was used. The error bars of all plots in this study display standard errors of the mean (SEM).

Predictions

It was expected that performance of finding the shortest possible route decreased with increasing number of targets of the TSP. This expectation was supported by two considerations. First, the number of route alternatives that had to be considered during planning increases with increasing TSP size. Second, working memory load is higher if more targets have to be memorized and dealt with. For sufficiently large numbers of targets, it will not be possible to simultaneously hold their positions in working memory. Thus, paths cannot be planned taking all targets into account. In the following the *specific predictions* for the different types of navigation tasks are summarized.

1. *NN-adequate/inadequate tasks* If participants employed the NN-strategy, they should find and navigate the optimal path when confronted with NN-adequate tasks. When confronted with NN-inadequate tasks, on the other hand, they should systematically fail to find the optimal paths (see Fig. 3). In other words, if participants employed the NN-strategy we expect better performance in NN-adequate than in NN-inadequate tasks.
2. *Cluster tasks (NN-ambiguous)* If participants applied the cluster-strategy, stating that they plan paths in order to visit as many targets as fast as possible, they should first visit the large rather than the small target cluster.

## Results

Neither the specific configuration of symbols on the 5 × 5 grid, nor the order of the symbols depicted on the *shopping list* influenced participants performance.

### *Experimental condition*

*Percentage above optimal (PAO)* On average, PAO was 5.86. Even for the largest TSPs (nine targets plus start place), participants produced less than 10 PAO (see Fig. 4). For three TSPs with nine targets plus start place, the PAO values for all 181,440 path alternatives was exemplarily calculated. Less than 0.08% of all path alternatives had PAO values equal or below the values produced by participants. The performance of the participants was thus remarkably above chance.

An ANOVA revealed a significant main effect of the number of targets [$F(5, 115.86) = 17.25$, $P < 0.001$], while no main effect for the type of navigation task [$F(2, 46.22) = 1.57$, $P = 0.22$] and no interaction [$F(10, 233.01) = 1.53$, $P = 0.13$] were found. PAO increased with increasing number of targets (Pearson's product-moment correlation: $r = 0.94$, $P < 0.01$). PAO did not differ between female and male participants [6.71 vs. 4.76%, *t*-test: $t(22) = 1.63$, $P = 0.12$].

*Found shortest route* On average participants found the shortest possible route in 47.3% of the trials. An ANOVA revealed a significant main effect for the number of targets [$F(5, 116.02) = 25.37$, $P < 0.001$] and the type of navigation task [$F(2, 46.27) = 79.09$, $P < 0.001$] as well as a significant interaction [$F(10, 233.01) = 6.88$, $P < 0.001$]. While a Pearson's product-moment correlation revealed only a marginally significant correlation between performance of finding the optimal route and the number of target locations ($r = -0.80$, $P = 0.06$), a significant difference was found between small TSPs (with 4–6 targets plus start place), and larger TSPs [with 7–9 targets plus start place; 32.7 vs. 61.6%, *t*-test: $t(23) = 8.89$, $P < 0.001$]. Performance of finding the optimal solution did not differ between female and male participants [44.13 vs. 50.87%, *t*-test: $t(22) = -1.43$, $P = 0.17$].

Performance in finding the optimal route did not differ between cluster tasks and NN-inadequate tasks [34.28 vs. 35.86%, *t*-test: $t(23) = 0.47$, $P = 0.64$], but differed both, between cluster tasks and NN-adequate tasks [34.28 vs. 72.32%, *t*-test: $t(23) = 10.88$, $P < 0.001$], and between

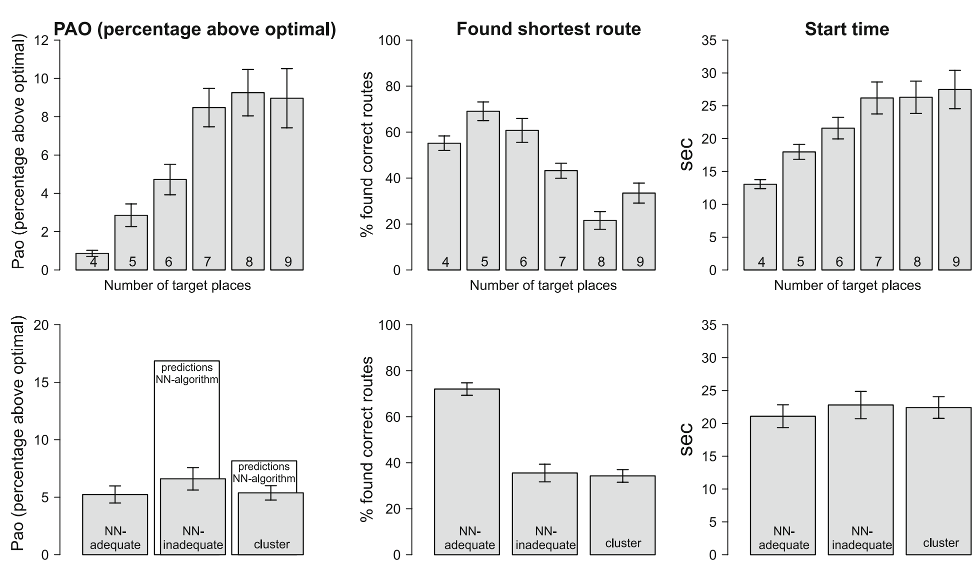


**Fig. 4** Results of Experiment 1

NN-adequate tasks and NN-inadequate tasks [72.32 vs. 35.86%, $t$-test: $t(23) = 10.19$, $P < 0.001$].

*Start time* On average start time was 22.10 s. An ANOVA revealed a significant main effect for the number of targets [$F(5, 115) = 24.02$, $P < 0.001$] while no main effect for type of navigation task [$F(2, 46) = 1.75$, $P = 0.19$] and no interaction [$F(10,230) = 1.21$, $df = 10$, $P = 0.29$] was found. Start time increased with increasing number of targets (Pearson's product-moment correlation: $r = 0.95$, $P < 0.01$). Start time did not differ between female and male participants [23.2 vs. 20.9 s, $t$-test: $t(22) = 0.63$, $P = 0.53$].

*Predictions of the NN-algorithm* The PAO predictions when using a NN-strategy were calculated for the different types of navigation tasks: for NN-adequate tasks it was obviously 0, for NN-inadequate tasks it was 16.92 and for cluster-tasks it was 8.13 (cluster tasks were NN-ambiguous: the NN strategy did not predict a single but multiple solutions as it was faced with situations in which the closest target locations were equidistant from its current position. PAO values were calculated by averaging over the different solutions predicted by the NN strategy). Participants' PAO for both, the cluster-tasks and the NN-inadequate tasks were significantly smaller than predicted by the NN-algorithm [cluster-tasks: 5.38 vs. 8.13%, $t$-test: $t(23) = 4.39$, $P < 0.001$; NN-inadequate tasks: 6.60 vs. 16.92%, $t$-test: $t(23) = 10.56$, $P < 0.001$]. For NN-adequate tasks, PAO was significantly higher than predicted by the NN-algorithm [5.24 vs. 0%: $t$-test: $t(23) = 7.05$, $P < 0.001$].

*Correlations between participants' start time and overshoot performance* Mean start time was negatively correlated with PAO ($r = -0.42$, $P = 0.04$), demonstrating that participants who took longer before initiating their trials showed better planning performance.

*Cluster tasks* In the cluster tasks the target locations were distributed in two distinct target clusters of unequal size. Overall, participants showed a significant preference to first visit the large cluster [59.02% vs. chance level (50%), $t$ test: $t(23) = 3.09$, $P < 0.01$].

## Discussion

Overall, PAO performance when solving the TSPs was remarkably good. On average, participants produced PAO values of less than 6. Even for the most complex navigation tasks with nine targets (plus start place), participants produced PAO values of ~10. The fact that less than 0.08% of all path alternatives of the largest TSPs tested produce PAO values below 10 emphasizes participants' remarkably good performance. With increasing TSP size, performance for finding the optimal solution decreased while start time increased. These results were expected for two reasons: (1) with increasing number of target locations the computational complexity of a TSP increases as more alternative solutions have to be taken into account; (2) the task of localizing and memorizing the positions of all target locations becomes more challenging as the number of target locations increases (i.e., SWM load increases).

*Types of navigation tasks* The most important result with respect to the planning strategies applied was that participants outperformed the NN-algorithm on NN-inadequate tasks and on cluster tasks. Together with the result that performance on NN-adequate tasks was significantly worse than predicted by the NN-algorithm, this clearly demonstrates that the NN-algorithm is not sufficient to explain human path planning in such navigational TSPs (for similar results in visual TSPs, see Graham et al., 2000). In cluster-tasks the target locations were distributed in two distinct target clusters of unequal size. Participants showed a preference to first visit the large target cluster as compared to the small target cluster. This result is in line with earlier work (Wiener et al., 2004) providing additional support for the cluster-strategy, stating that participants plan their routes in order to visit as many targets as fast as possible (for similar results in vervet monkeys see Cramer & Gallistel, 1997). While for both, path planning performance (PAO) and start time, no significant differences could be found between the three types of navigation tasks, performance of finding the shortest route was almost twice as good for NN-adequate tasks than for NN-inadequate tasks and cluster-routes. This dissociation between planning performance (PAO) and performance of finding the optimal route suggests that many errors on cluster- and NN-inadequate tasks were insignificant with respect to the resulting PAO values.

*Interviews with participants* Further insights into planning strategies came from informal interviews with participants after the experiments. Most of them reported to have applied one of two strategies when faced with larger TSPs: (1) Participants subdivided the 25 locations into a (differing) number of regions. During planning, they assigned the actual target locations to these regions and planned a coarse route on that region level. Such coarse routes are simple and easily remembered and a fine-detailed plan can be created by inserting close-by target locations during navigation; (2) Participants first selected a subset of target locations depicted on the shopping list according to some criteria, for example, color. They then planned a coarse route taking into account only this subset. Again, this route plan is simple and easily remembered and can be refined either before or during navigation by inserting the missing target locations into the route. Both of the reported navigation strategies follow essentially the same logic: they simplify the planning task by applying a hierarchical planning scheme. First, a coarse and simple path plan is generated on basis of an abstraction of the environment or the planning task itself. This path plan is then refined during navigation by inserting target locations.

## Experiment 2

### Motivation

Experiment 2 served two main purposes: (1) to test the region based planning strategy reported by participants in informal interviews after Experiment 1; (2) to test for the role and impact of different memory systems for spatial problem solving and optimization.

Informal interviews with participants in Experiment 1 suggested that one planning strategy was based on participants' regionalizations of the environment. If based on regions, path planning becomes a hierarchical process. First, a coarse route plan is generated on the level of the regions exclusively. This plan is then refined during navigation. Such a planning scheme states that first all target locations in one region are visited before the next region is entered. Experiment 2 tested this region-based planning strategy. For this, the environment was subdivided into different objective regions and participants solved similar TSPs as in Experiment 1.

Results from Experiment 1 furthermore suggested that capacity limits of SWM had a crucial impact on both, planning performance and start time. Here we specifically tested for the role and impact of both SWM and spatial LTM for path planning by systematically manipulating memory demands between the experimental conditions.

### Materials and methods

#### *The experimental setup*

The general setup of Experiment 2 was identical to Experiment 1, but differed in the arrangement of the symbols on the 5 × 5 grid. Symbols of equal color were neighboring each other, thus creating five clearly distinct regions in the environment (see Fig. 5). Participants reported to have perceived the regions very well and that they also used them during path planning. As in Experiment 1 we controlled for effects of the specific symbol-configuration by creating two versions of the setup that differed in the arrangement of the symbols. In each condition, half of the participants conducted the experiment in one configuration, the other half conducted the experiment in the alternative configuration.

#### *Procedure*

Each participant solved 36 TSPs consisting of a start location plus 4, 5, 6, 7, 8, or 9 target locations (TSP sizes therefore ranged between 5 and 10; for a detailed description of all routes see Table 2). The TSPs were presented in random order. The exact procedure of presenting target

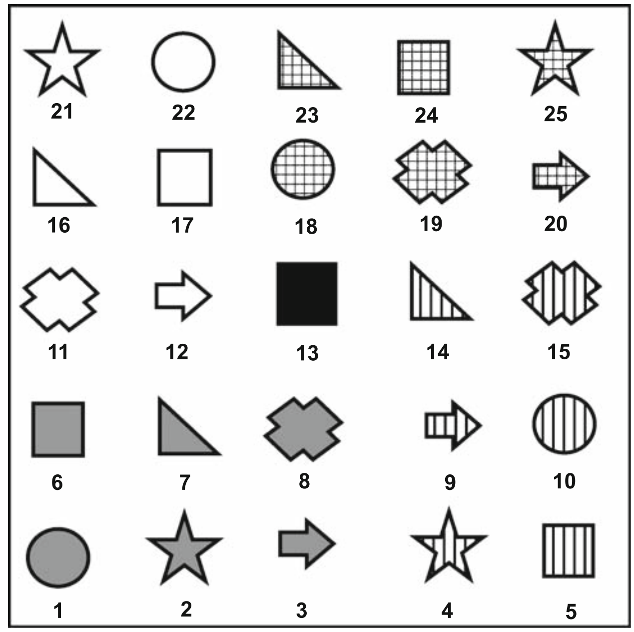


**Fig. 5** Setup of Experiment 2. By arranging the symbols according to their color, five distinct regions were generated. The image is adapted to represent the original colors of the symbols (black, yellow, red, green, blue) in *black-and-white*

locations and marking visited locations during navigation differed between experimental conditions. It is explained below.

#### *Experimental conditions*

Experiment 2 featured three experimental conditions that differed with respect to how target locations were presented and whether or not the symbols were visible during the experiment:

*Condition A (no memory condition)* Goal locations were directly marked in the environment rather than being depicted on a *shopping list*. For each trial, the experimenter marked each target location with a black marker while participants faced a wall. During navigation participants collected the markers.

*Condition B (SWM condition)* With respect to presenting the target locations, this condition was identical to Experiment 1. Participants were given a *shopping list*, depicting the symbols defining the start and the target locations for each TSP. They localized the relevant target locations using this shopping list. In contrast to condition A, this condition required that participants maintained a temporary representation of the spatial arrangement of the target locations, a SWM, to plan their path. As in Experiment 1 participants marked the visited places with little black markers.

**Table 2** The table lists all navigation tasks of Experiment 2

| Navigation task | Number of target places | Start place (target places) |
|---|---|---|
| RS-adequate | 4 | 6 (9,5,10,19), 22 (12,3,13,20), 6 (7,13,15,17) |
| | 5 | 22 (12,2,4,5,18), 17 (11,13,4,15,19), 25 (13,7,4,5,9) |
| | 6 | 13 (22,20,14,9,5,7), 25 (13,7,3,4,5,9), 2 (17,21,22,23,19,18) |
| | 7 | 9 (8,1,12,11,21,17,18), 11 (6,8,14,10,15,25,19), 1 (16,12,13,18,24,14,9) |
| | 8 | 4 (9,14,19,23,16,17,13,2), 10 (9,8,13,12,11,16,21,19), 20 (18,17,21,16,7,13,9,10) |
| | 9 | 15 (9,3,7,13,12,11,16,22,24), 9 (4,5,10,14,19,25,24,18,8), 10 (4,8,2,1,16,13,18,24,20) |
| RS-inadequate | 4 | 23 (18,14,4,20), 15 (12,6,11,22), 5 (15,24,14,7) |
| | 5 | 20 (24,14,7,4,10), 10 (18,22,11,6,12), 1 (3,5,15,20,7) |
| | 6 | 12 (21,19,13,8,4,6), 20 (17,21,16,11,7,12), 10 (18,22,16,11,6,12) |
| | 7 | 6 (1,3,9,5,10,20,14), 21 (6,12,13,8,4,14,19), 20 (19,22,13,12,2,8,9) |
| | 8 | 11 (13,14,20,15,4,8,2,1), 25 (20,15,14,9,4,3,2,18), 1 (2,3,4,10,19,14,8,11) |
| | 9 | 20 (19,24,23,18,14,9,3,4,10), 6 (2,8,4,5,20,13,18,22,16), 11 (22,24,20,15,14,13,9,3,7) |

The start place is followed by the target locations (*in brackets*). The numbers correspond to the place numbers in the schematized drawing of the experimental environment (see Fig. 5)

*Condition C (SWM and LTM condition)* In condition C, the symbols of all 25 places were hidden under cardboard covers. Participants learned the positions of the symbols in a training phase prior to the test phase (explained below). In the test-phase, they were given a *shopping list* for each TSP (see above). Participants had to retrieve the exact locations of the targets from LTM and had to maintain a temporary representation of these positions (SWM) during path planning. They navigated to the target locations and removed the cardboard cover in order to mark the places already visited. Errors (i.e., if cardboards were removed although the corresponding place did not belong to the target locations) were recorded.

To control for the influence of the specific order of symbols depicted on the *shopping list*, two versions of each shopping list were generated. Half of the participants received one version of the shopping lists, while the other half received the other version of the shopping lists.

*Training procedure of condition C* The training phase consisted of three sessions. First, participants were given 5 min to study the layout of the environment without the cardboard covers. Subsequently, the symbols were covered and participants received 13 shopping lists, depicting two symbols (each of the 25 symbols was presented at least once). Their task was to find both symbols. In the third phase, participants received nine shopping lists, each of which depicted three symbols. Their task was to find them. If participants solved the last five tasks without error, they entered the test phase; if not, the covers were removed and participants were allowed to study the layout for another 5 min.

*The navigation tasks*

Navigation tasks could be assigned to one of two types, the *Region-Strategy adequate tasks (RS-adequate tasks)* and the *Region-Strategy-inadequate tasks (RS-inadequate tasks).*

1. *Region-Strategy-adequate tasks (RS-adequate)* The optimal solution for RS-adequate tasks requires to first visit all target locations in one region before moving to the next region. This is in line with predictions from a region-based planning approach (see Fig. 6). Thus, employing a region-based strategy could result in the optimal route.
2. *Region-Strategy-inadequate tasks (RS-inadequate)* Employing a region-based planning strategy on RS-inadequate tasks will systematically lead to sub-optimal paths (see Fig. 6), as navigating the optimal solution requires to leave a region and to re-enter it later. Furthermore, if routes are planned on the region level, the resulting paths should systematically cross fewer region boundaries as compared to the optimal path.

It is important to note that the spatial configuration of start location and target locations was always identical for pairs of two TSPs, one of which was from the RS-adequate group and one of which was from the RS-inadequate group (see Fig. 6). Any differences in behavior between the RS-adequate and the RS-inadequate group could thus be clearly attributed the the region characteristics of the navigation

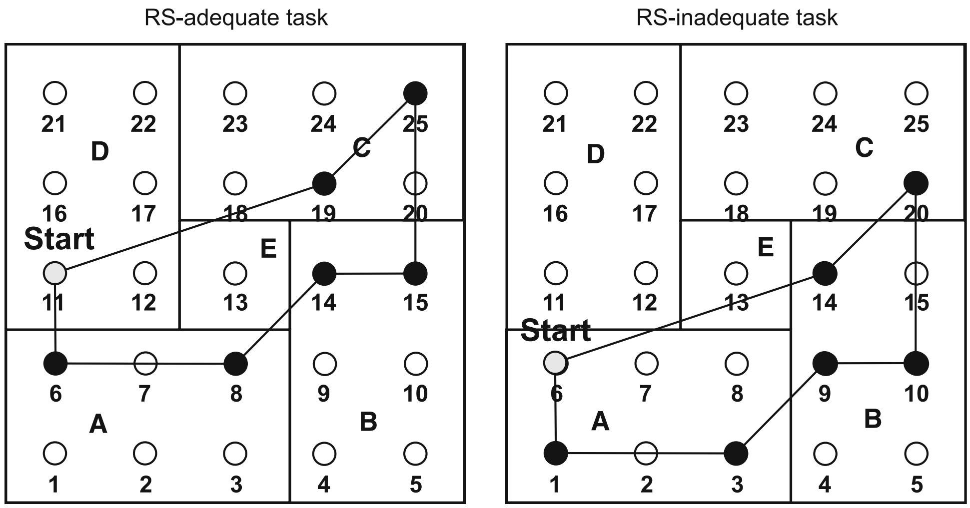


**Fig. 6** Examples for a *RS-adequate task* (*left*) and a *RS-inadequate task* (*right*). *Grey circles* represent the starting places, *solid black circles* represent target locations, and the *black line* represents the optimal solution. Note that these two tasks are identical with respect to spatial configuration of start and target locations, and therefore also with respect to the optimal solution. Navigating optimal solutions in RS-inadequate tasks requires to leave one region and to re-enter it later, which is not in line with a region-based strategy. The two different types of navigation tasks used in Experiment 2 are generated by simply mirroring and/or shifting the configuration of start and target locations; Analysis on the region level (each region is is represented by a *capital-letter*): on the region level the RS-adequate route is described as D-A-A-B-B-C-C-D while the RS-inadequate route is described as A-A-A-B-B-C-B-A

tasks. Participants solved the same 36 navigation tasks in all three conditions.

*Participants*

Altogether, 72 participants participated in Experiment 2. In each condition 24 participants were tested (condition A: 12 women, mean age: 25.21 years; condition B: 15 women, mean age: 25.13 years; condition C: 14 women, mean age: 26.13 years). Participants were naive with respect to the specific hypotheses of the experiments. They were mostly university students and were paid 8 Euro an hour.

*Analysis*

In addition to the analysis on the level of single places described in Experiment 1, the chosen paths were also analyzed on the level of regions: for example, the left path displayed in Fig. 6 was described on the place level as follows: 11-6-8-14-15-25-19-11. On the region level the same path was represented as D-A-A-B-B-C-C-D. From this region representation, the number of region crossings was calculated for every chosen path as well as for the corresponding optimal solution. Furthermore, by comparing the region-representation of a traveled path with the region-representation of the optimal solution (optimal region route), errors at the region level were analyzed independent of errors at the place level. In other words, a chosen path could be different from the optimal solution on the place level, because the order in which places within one region were visited was suboptimal. On the region level, however, this route would be indistinguishable from the optimal solution.

Due to the large number of main effects and interactions tested in this experiment, an $\alpha$ level of 0.01 was used.

Predictions

*Region-based planning strategies* Employing the proposed region-based path planning strategy will prevent participants from finding the optimal solution when navigating RS-inadequate tasks. It was therefore expected that participants showed decreased performance in finding the optimal solution on RS-inadequate tasks as compared to RS-adequate tasks. More specifically, if participants employed a region-based strategy, it was expected that they produce more errors on the region-level (see "Analysis") when navigating RS-inadequate tasks as compared to RS-adequate tasks: on RS-inadequate tasks the region strategy will lead to fewer region crossings as compared to the optimal solution. Performance differences in terms of PAO were not necessarily expected between RS-adequate and RS-inadequate tasks, as (1) following a region strategy on RS-inadequate tasks not necessarily results in large detours, and (2) as we expected that participants would often navigate suboptimal solutions in both types of navigation tasks (errors can be made on the region level, but also on the place level within a single region).

*Memory demands* With increasing memory demands, fewer cognitive resources are available to be attributed to the actual planning task. In general, we therefore expected that the dependency on simplifying planning strategies increased with increasing memory demands between conditions. Following the same logic, we expected performance differences between conditions. Specifically, performance should be best in condition A in which all target locations were marked in the environment and participants did not have to deal with memory related constraints. Condition B and condition C both required maintaining the exact target positions in a capacity limited SWM after localization. We therefore expected performance decreases with increasing TSPs size. A priori, we did not expect performance differences between condition B and condition C. Although information about the exact target positions had to be retrieved from LTM in condition C, there is no obvious reason why planning performance per se should be influenced depending on the whether information about target positions comes from LTM or directly from visual perception.

## Results

Only correct trials (i.e., trials in which participants made no errors in terms of visiting places that were not target locations) entered the following analysis. In condition A (no memory) 1.8% of the trials were discarded from the analysis, in condition B we discarded 5.3% and in condition C we discarded 16.8% of the trials. Neither the specific configuration of symbols on the 5 × 5 grid, nor the order of the symbols depicted on the *shopping list* influenced participants performance.

*PAO* An ANOVA [within-participant factors: number of targets (4–9) and type (RS-adequate, RS-inadequate), between-participant factor: condition [A,B,C]], revealed significant main effects of number of targets [$F(5, 69) = 17.78$, $P < 0.001$] and condition [$F(2, 69) = 10.32$, $P < 0.001$], while no main effect for type [$F(1, 69) = 0.27$, $P = 0.61$] was found. Only one of the two-way interactions, number of targets x condition, revealed a significant effect [$F(10, 69) = 4.63$, $P = 0.01$]. Specifically, PAO for condition C (working memory and long-term memory) was higher than for condition A and condition B (post hoc tests), and PAO increased with increasing TSP size (see Fig. 7).

To separate the influence of working memory, the TSPs were dichotomized according to their size resulting in small TSPs (with 4–6 targets plus start place) and larger TSPs (with 7–9 targets plus start place). Data for condition A (no memory) and condition B (working memory) were reanalyzed: an ANOVA [within-participant factor: TSP size (small, large) and type (RS-adequate, RS-inadequate)], between-participant factor: condition [A,B], revealed a main effect of TSP size [$F(1,46) = 23.66$, $P < 0.001$] as well as a significant interaction between size and condition

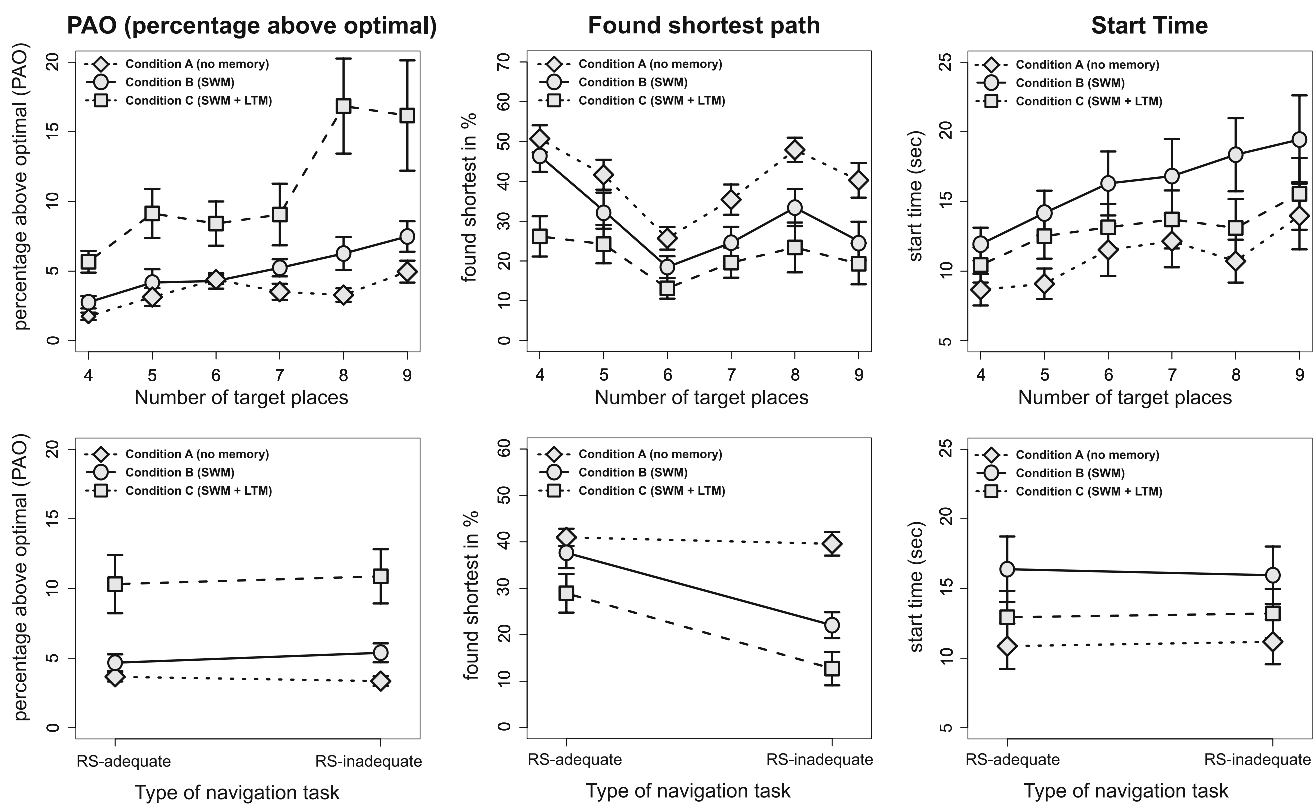


**Fig. 7** Results of Experiment 2

[$F(1, 46) = 7.13$, $P = 0.01$]. Specifically, PAO performance for small TSPs did not differ between conditions A and B (3.09 vs. 3.74 PAO, *t*-test: $t(46) = 1.15$, $P = 0.26$), while PAO performance differed for larger TSPs (3.92 vs. 6.37 PAO, *t*-test: $t(46) = 2.99$, $P < 0.01$). The effect of condition did not withstand $\alpha$ level correction [$F(1, 46) = 6.17$, $P = 0.02$].

PAO performance did not differ between female and male participants (6.85 vs. 5.67 PAO: $t(70) = -0.74$, $P = 0.45$).

*Found shortest path* An ANOVA [within-participant factors: number of targets (4–9) and type (RS-adequate, RS-inadequate), between-participant factor: condition [A,B,C]], revealed significant main effects of number of targets [$F(5, 69) = 14.84$, $P < 0.001$], type [$F(1, 69) = 41.15$, $P < 0.001$], and condition [$F(2, 69) = 12.35$, $P < 0.001$]. Most importantly, performance of finding the optimal route decreased with increasing memory demands (post hoc-tests revealed significant differences between all three conditions), and was better in RS-adequate than in RS-inadequate tasks. Only one of the two-way interactions, type x condition, revealed a significant effect [$F(2, 69) = 7.82$, $P = 0.001$]. Specifically, performance of finding the optimal solution was independent of the type of navigation task in condition A (no memory condition) only. In both, condition B and condition C performance was worse in the RS-inadequate type than in the RS-adequate type (see Fig. 7).

*Start time* Generally, start time increased with increasing TSP size. An ANOVA [within-participant factors: number of targets (4–9) and type (RS-adequate, rs-inadequate), between-participant factor: condition [A,B,C]], revealed a significant main effect of number of targets [$F(5,69) = 21.07$, $P < 0.001$], while no main effects of condition [$F(2, 69) = 1.89$, $P = 0.16$] or type [$F(1, 69) = 0.03$, $P = 0.86$] were observed.

*Optimal region route* A repeated measures ANOVA [within-participant factors: number of targets (4–9) and type (RS-adequate, RS-inadequate), between-participant factor: condition [A,B,C]], revealed significant main effects of number of targets [$F(5, 69) = 3.96$, $P < 0.01$], type [$F(1, 69) = 220.83$, $P < 0.001$] and condition [$F(2, 69) = 15.55$, $P < 0.001$]. Most importantly, participants found the optimal region route more frequently on RS-adequate than on RS-inadequate tasks (see Fig. 8). With increasing memory demands between conditions, performance of finding the optimal region route decreased (post hoc-tests revealed significant difference between all three conditions).

*Region crossings* When solving the RS-inadequate navigation tasks, participants crossed less region boundaries than would have been expected for optimal solutions. On average, 4.11 region transitions were made on RS-inadequate tasks, which is a reduction by 0.61 from the expected 4.72 region transition for optimal solutions (4.11 vs 4.72, *t*-test: $t(23) = 14.69$, $P < 0.001$, see Fig. 8). On RS-adequate routes, on the other hand, 4.37 region transitions were made on average, which is an increase of 0.15 from the expected 4.22 region transitions for optimal solutions (4.37 vs 4.22, *t*-test: $t(23) = 6.36$, $P < 0.001$, see Fig. 8).

## Discussion

*Path planning strategies* The region-based planning strategy states that first a coarse path is planned on the region-level that is refined during navigation by including close target locations. To test this hypothesis, two types of navigation tasks were generated, Region-Strategy-adequate tasks (RS-adequate) and Region-Strategy-inadequate tasks (RS-inadequate). As employing region-based strategies prevented participants from finding the optimal solution on RS-inadequate tasks (see "The navigation tasks") we expected worse performance on RS-inadequate tasks than in RS-adequate tasks. This prediction was in line with results from condition B and condition C, but not with results from condition A, in which performance of finding the optimal solution was independent of the type of navigation

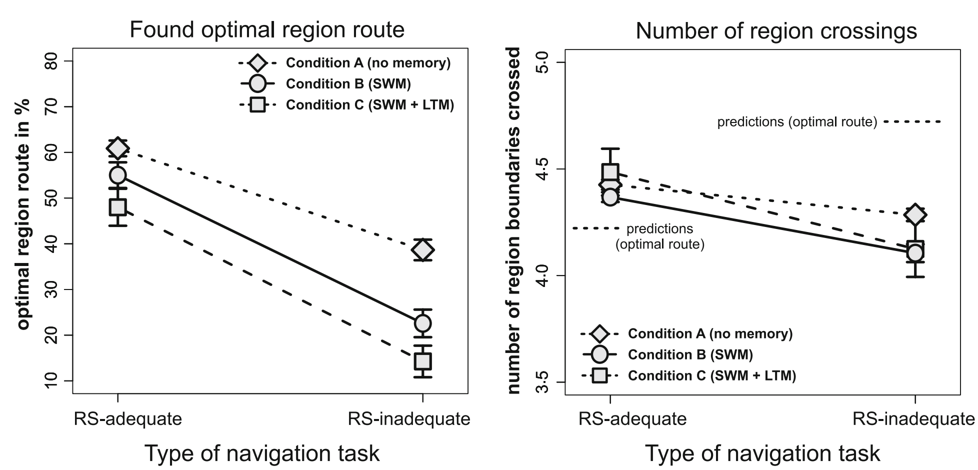


**Fig. 8** Results of the region based analysis of Experiment 2

task. Furthermore, it was predicted that participants produce more errors on the region level for RS-inadequate than for RS-adequate tasks, when compared to the optimal solution. While this prediction was met for all three experimental conditions, performance in finding the optimal path on the region level was best for condition A (no memory condition), followed by condition B (SWM), and condition C (SWM and LTM condition). Apparently, performance of finding the optimal path on the region level in condition A was not as strongly influenced by the type of navigation task as for condition B and condition C. Finally, for all three conditions we found that, when navigating RS-inadequate tasks, participants crossed fewer region boundaries as compared to the optimal solution. Taken together, these results demonstrate that participants minimized the number of region crossings during path planning and navigation as predicted by the region-based planning strategy (cf. Wiener & Mallot, 2003; Wiener et al., 2004). While this statement holds true for all three experimental conditions, the influence of the region-based strategy differed between experimental conditions. Specifically, it was weakest in condition A (no memory condition).

Hierarchical planning schemes have already been suggested for visual versions of the TSP (e.g., Graham et al., 2000; Vickers et al., 2003a; Kong & Schunn, 2007). All these hierarchical problem solving methods, including the described region-based approach, are essentially based on clustering or chunking of (spatial) information to generate an abstraction of the original problem. First a coarse and simple solution for the problem is found using this abstraction. This solution is later refined. By these means a large number of suboptimal solutions are usually ruled out, the search space is significantly reduced, which consequently reduces the computational complexity of the task. Chunking processes are known to be common, if not universal, principles of human perception, learning, and cognition (e.g., Gobet et al., 2001). Despite the systematic differences between visual and navigational TSPs, pointed out in the "Introduction", the general problem solving heuristics allowing to handle complex and computationally expensive spatial optimization tasks may in fact reduce cognitive effort in similar ways.

*Planning performance and memory demands* Generally, Experiment 2 rendered similar results as Experiment 1. Planning performance (both PAO and percentage of trials in which optimal solution was found) decreased with increasing TSP size, while start time increased. Overall, planning performance was best in condition A, requiring no memory, followed by condition B, requiring SWM, followed by condition C, requiring both, SWM and spatial LTM. As only correct trials entered the analysis (i.e., trials in which participants made no errors in terms of visiting places that were not target locations), differences in planning performance between conditions were particularly interesting: they cannot be attributed to detours and corrections caused by visiting wrong places. In other words, they cannot be attributed to errors in memory. Rather, performance differences demonstrate that the planning process itself differed or became less efficient with increasing memory demands.

A comparison of condition A (no memory condition) and condition B (SWM condition), for example, highlights the crucial role of SWM for path planning. For small TSPs with up to six targets, supposedly not reaching capacity limits of SWM, planning performance did not differ between conditions A and B. On larger TSPs (7–9 targets plus start place), however, participants showed significantly better performance if they did not have to remember the exact locations of the targets in SWM, but if these were directly marked in the environment. The crucial role of SWM for path planning is further corroborated by the dramatic increase in PAO (performance decrease) for larger TSPs as compared to smaller TSPs in condition B.

One possible reason for these differences is that, according to memory demands, participants used different planning strategies. One might argue that condition A (no memory condition) facilitates the usage of global planning mechanisms, taking all targets into account at the same time. Target places were marked in the environment and were therefore directly visually accessible. To this extent, condition A closely resembles visual TSPs (see "Introduction") for which it has been suggested that solution processes correspond to organizing principles of visual perception (e.g., MacGregor & Ormerod, 1996; Graham et al., 2000). In condition B and condition C, on the other hand, participants had to localize the targets in the environment by using the shopping lists (condition B) or by using shopping list and spatial LTM (condition C). In both cases, participants had to maintain a temporary representation of the exact positions of the targets in SWM after localization. Obviously, global planning strategies are not feasible in these conditions if the number of targets to be considered exceeds SWM capacity limits. Here path planning strategies that reduce SWM load appear more suitable. One such strategy is the region-based planning heuristic discussed above. Taken together, this suggests that with increasing memory demands, participants relied more strongly on simplifying heuristics such as the region-based planning heuristic to overcome memory related constraints. This prediction was met: the influence of the region-based planning strategy was weakest in condition A (no memory condition).

In addition to working memory, spatial LTM had a severe impact on optimization performance, particularly on the quality of the chosen path. Whenever spatial information had to be retrieved from LTM, the planning process became less efficient. This was surprising to some extend, as one might argue that for the two conditions in which spatial locations had to be maintained in working memory (conditions B and C) no performance difference had to be predicted: particularly for small TSPs not exceeding capacity limits of working memory, it should be irrelevant whether spatial information was retrieved from LTM (condition C) or from visual perception (condition B). However, even for the smallest TSPs tested (with start place plus four targets), clear performance differences were found. One possible explanation for the reduced planning efficiency in condition C as compared to condition B is that the effort of retrieving positional information from LTM interfered with SWM: assuming a limited pool of distributed resources, fewer resources would thus be available for the integration and storage of the positional information in SWM. This would consequently result in a decreased SWM capacity. This interpretation would also explain the start time results of Experiment 2: at first glance it appeared counter-intuitive that start times for condition B (SWM) were longer than for condition C (SWM + LTM). However, if SWM capacity was reduced in condition C as compared to condition B, fewer locations could be encoded in SWM and, therefore, less time was spend for encoding. Additionally, if fewer locations are taken into account during an initial planning step, less time is required for planning.

## Conclusion

In this work we studied cognitive strategies and processes involved in complex spatial problem solving using navigational versions of the well-known TSP. In Experiment 1 we demonstrated that participants outperformed the simple NN strategy, that has been suggested to be involved in human and animal path planning (e.g., Gärling & Gärling, 1988; Bures, Buresova, & Nerad, 1992). This result demonstrates that the NN-strategy is not sufficient to explain planning behavior for complex TSPs. We also demonstrated that participants combined neighboring targets to clusters and that they preferred to first visit large rather than small clusters (cf. Gallistel & Cramer, 1996; Wiener et al., 2004). In Experiment 2 we demonstrated that participants relied on a region-based planning strategy, stating that they divided the environment into different regions, that they assigned the targets of the TSP to these regions, and that they first planned a coarse path on the region level which was then refined during navigation (see also Wiener & Mallot, 2003; Wiener et al., 2004). The described hierarchical planning scheme constitutes a planning heuristic as it reduces computational effort and memory load during path planning while still resulting in reasonably short solutions (for a recent review on heuristics, see Shah & Oppenheimer, 2008). Finally, we addressed the relevance and impact of different memory systems (SWM and spatial LTM) for path planning by varying task difficulty (TSP size) and memory demands between experimental conditions. Path planning performance systematically decreased with increasing TSP size and with increasing memory demands. At the same time, the dependence on the region-based planning heuristic increased. However, it was evident in all conditions, suggesting that such hierarchical planning heuristics represent general problem solving strategies in navigational TSPs.

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