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RESEARCH REPORT

The verbalization of multiple strategies in a variant of the traveling salesperson problem

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Abstract What kinds of strategies do humans employ when confronted with a complex spatial task, and how do they verbalize these strategies? Previous research concerned with the well-known traveling salesperson problem (TSP) typically aimed at the identification of a generally applicable heuristics that adequately represents human behavior in relation to the abstract task of combining points. This paper adopts a novel perspective in two respects. On the one hand, it addresses the strategies people employ when confronted with a more complex task, involving distractors and feature information rather than identical points. On the other hand, retrospective linguistic representations of the strategies used are analyzed in relation to the behavioral data, using discourse analytic methods. Results show that both the behavioral results and the verbalizations point to a range of strategies related to those proposed for solving abstract TSPs. However, in contrast to earlier accounts in the literature, the participants employ a repertory of multi-faceted strategies and planning processes, simplifying and structuring the problem space across subtasks and processes in flexible ways. These findings provide further insight into the nature of human strategies in spatial problem solving tasks and their retrospective verbalization, highlighting how procedures generally known in the literature may be adapted to more complex tasks, and how they may be verbalized spontaneously.

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Introduction

Imagine planning a trip through Europe, using a map and a list of goals that you want to visit. You wish to start from, and return to, your home town; and you wish to avoid detours in order to save time and fuel. How do you plan the tour? Will you mentally compute all possible trajectories that connect all targets and then choose the shortest solution, or will you draw on a particular strategy for optimizing your trajectory?

This scenario can be conceived of as an instance of the well-known "traveling salesperson problem" (TSP). Here the aim is to find the shortest way of connecting a number of places or target points to each other before returning to the starting position. This problem is interesting from various perspectives within the area of Cognitive Science for a number of reasons. First, it represents a computationally complex (often called "NP-hard") problem for which a generally applicable optimal algorithm is not available to date, producing the best solution within a reasonable amount of time (e.g., Golden et al. 1980; Applegate et al. 2007). Second, humans are known to solve this problem efficiently and with very good, if not exactly optimal, results even if a high number of target points need to be connected (Gärling et al. 1986; MacGregor et al. 2000). To achieve this, they mentally reduce the problem complexity by employing heuristics as a cognitive simplifying strategy, rather than comparing all possible solutions (cf. Todd and Gigerenzer 2000) and then choosing the shortest path. The precise natures of these heuristics have been subject to a range of controversies in the literature during the past decades (see next section). Third, the problem is closely related to some problems well-known in every-day life, including areas of focal interest within wayfinding research (Golledge 1999; Reineking et al. 2008). Here, the main question concerns how the original abstract, and mostly computer-based, problem relates to actual human planning and navigation behavior, and how the strategies identified for this particular spatial task can be generalized to other problems involving human spatial cognition. This paper sets out to shed further light on these issues by enriching the original abstract task with salient feature information, and gathering further information about human strategies by investigating the associated language using discourse-analytic methods. Such additional knowledge about human cognitive strategies supports the development of suitable algorithms for solving the TSP efficiently.

TSP strategies

Previous findings on TSP strategies have identified a range of candidates considered as competing for the best heuristic or algorithm that adequately accounts for empirical findings on humans' solutions. MacGregor et al. (2000) propose the *convex hull method* in which an elastic band can be visualized that encompasses all boundary dots; remaining dots can be inserted into the tour at those points where the band is closest. The *crossing avoidance* strategy (van Rooij et al. 2003), according to which humans attempt to avoid line crossings, yields similar results. Some of the participants of a study by Vickers et al. (2001) report using a *circular* strategy or avoiding self intersections; overall, their behavioral TSP results were strikingly similar to another group who were explicitly asked to find an *aesthetically pleasing* path. In another account (based on a navigational TSP variant), humans start out by conceptualizing the trajectory first on a coarse level of precision, for example, by imagining the path as an approximate circle or other kind of regular pattern. Then they decide on the specific steps to take by inserting detours from the general trajectory (Tenbrink and Wiener 2007). We call this strategy the *trajectory-based* method. These strategies, though superficially similar and resulting in related behavioral outcomes, differ with respect to the associated thought processes, and therefore the ensuing verbalizations should be distinguishable.

Other approaches center around spatial vicinity and clustering procedures. Gärling et al. (1986) suggested that humans try to minimize distances locally in a sequential process. More recently, Best and Simon (2000) showed that the *Nearest Neighbor* algorithm in fact explains a large portion of human behavioral data; combining this strategy in a two-stage process with an exploration strategy (involving rotation around the center of mass) yields an even better match with human performance. Other approaches support a *hierarchical nearest neighbor* (Vickers et al. 2003) or a *hierarchical pyramid* algorithm (Graham et al. 2000). These strategies involve focusing on clusters of targets that are situated particularly close to each other, and subsequently combining these clusters in a tour.

So far, there is no indication in the literature as to how the cluster-based aspects relate to the trajectory-based processes; also, the possibility of a high relevance of purely perceptual processes raised by MacGregor and Ormerod (1996) remains a matter of controversy. Are these findings to be understood as competing theories concerning the ways in which humans solve these tasks, or could they represent the range of variability in cognitive processes that are generally available to humans when solving TSP tasks? Dry et al. (2006) recently speculated that humans combine specific kinds of "bottom-up" *perceptual* processes with "top-down" *cognitive* processes. This idea is consistent with findings on the critical role of apperception in other areas of problem solving (Helfenstein and Saariluoma 2007). Possibly, the apparently competing theories may be reconciled in this way; however, so far no specific evidence for such a combination in the TSP has been forthcoming.

In this paper we focus on a subset of the range of strategies represented in the earlier literature. Wiener et al. (2008b) introduce a specific TSP-like scenario which highlights the usage of the *cluster strategy,* the *NN (nearest neighbor) strategy,* and the *region-based strategy* (specified below), using real navigation within a room. Although this does not exclude the usage of further strategies during the problem solving process, the design allows for assessing the impact of each of these particular strategy choices on performance. This design is here employed in a paper-and-pencil variant.

Identifying cognitive strategies

There is a range of ways of identifying cognitive strategies that humans may employ when confronted with a complex spatial task. One well-established method is to design experiments that specifically address the relevance of a particular aspect or strategy, based on the idea that certain cognitive processes and strategies may lead to different outcomes with respect to tasks that share a particular feature (e.g., concerning the spatial configuration) than to other tasks that do not share this feature (Gärling 1989; MacGregor and Ormerod 1996; Vickers et al. 2003; Wiener et al. 2008b). Another method is to formulate hypotheses based on behavioral data and then model the strategies computationally, predicting their outcome when applied to a particular task. This approach has proved to be useful and informative in many respects, also concerning TSP strategies (e.g., MacGregor et al. 2000; Graham et al. 2000). However, to date no algorithm is available that adequately accounts for humans' performance in the TSP (Dry et al. 2006). Thirdly, one may ask humans to report how they solved the task (e.g., Polivanova 1974; Vickers et al. 2001). The amount of insight gained from this approach depends on the degree to which the problem solvers are aware of their cognitive processes and strategies. Ericsson and Simon (1984) examine the utility of various kinds of verbal protocols in detail; they found that while thinking-aloud protocols as well as carefully elicited retrospective reports provide reliable insights into actual cognitive processes derived from working memory, questionnaires are more suitable to elicit consciously edited and generalized higher-level ideas. The relationship of the cognitive processes to the linguistic results will depend heavily on the formulation of the questions posed to the problem solver.

These three methods complement each other in useful ways and have often been combined in research. Nevertheless, there seems to be no general agreement concerning the nature of the underlying strategies humans employ in solving TSP tasks. One problem in the identification of TSP strategies lies in the fact that human thinking may not be adequately accounted for by any one-dimensional model. Evidence from other fields of spatial cognition (Loomis et al. 1999; Allen 1999) and cognitive processing (Payne et al. 1992; Helfenstein and Saariluoma 2007) indicates that humans attend to a broad variety of aspects from various perceptual and conceptual sources while solving spatial tasks on various scales. The current literature concerned with the TSP does not match well with such insights on cognitive complexity, in spite of the fact that a well-received early study investigating verbal protocols of a related naturalistic planning task already pointed to highly complex planning processes (Hayes-Roth and Hayes-Roth 1979).

One reason for this discrepancy lies in the fact that most empirical TSP work so far has been restricted to abstract versions in which the available perceptual input consists of identical points, all of which need to be connected. This kind of visual problem stands in sharp contrast to highly complex real world navigation tasks involving various kinds of goals and distractors, i.e., elements that are perceptually equally salient as the intended goals but that are not relevant for the task (see Wiener and Tenbrink 2008 for an overview). When planning a trip through Europe, for instance, the map will show a complicated array of salient cities and streets; thus, focussing on the intended goals in order to identify a good trajectory may be cognitively quite demanding. Nevertheless, human solutions to such abstract TSP versions are regarded as being related to more general spatial thinking with good reason; everyday navigation tasks such as shopping or sightseeing round trips share more than one feature with the abstract TSP (Gärling et al. 1986), and some of the hypotheses targeted in abstract TSPs originated from the early naturalistic studies. It remains unclear, however, to what extent salient contextual features that are ubiquitous in the real world, such as the presence of multiple entities in the world, as well as differences in color, shape, size, attractiveness, safety, simplicity of the route, and other salient factors may systematically influence human strategies. With respect to finding the way to a single goal, Golledge (1997) proposes a range of criteria used in path selection, and Wiener et al. (2008a) find that route choices may depend on the circumstances of planning.

While our scenario in the present endeavour remains abstract and in this respect closer to typical experimental TSP settings than to real life, we purposely integrate two additional features to enrich the scenario: the presence of distractors and distinct object features (color and shape). Distractors have been shown to play a distinct role in spatial memory (e.g., Janzen and Weststeijn 2007). Using irrelevant elements in a TSP variant naturally entails higher demands on visual search as well as memory processes (Williams et al. 2005), which interact with the actual TSP planning processes in hitherto unknown ways. Color and shape are used as prominent object features to distinguish target from distractor objects. Color in particular is known as a biologically significant feature that is accounted for even if irrelevant for the current situation (Elliot and Maier 2007; Liqiang et al. 2004). How do humans then deal with the relative significance and relevance of spatial proximity, shared colors, and other involved object features? Do strategies change if spatial goals are arranged according to a salient object feature such as color, suggesting a clustering process that may not as such be supportive of the given problem? Then color as a salient and (at least for the identification process) *task-relevant* feature is associated with (*task-irrelevant*) spatial connectivity, enhancing visual search processes based on color (Kim and Cave 1999). How such features are conceptually intertwined is an open question. While the cognitive model proposed by Hayes-Roth and Hayes-Roth (1979) integrates planning processes at several levels including the identification of goal locations among distractors, their task scenario is too remote from abstract TSP settings to enable direct inferences for the generation of cognitively adequate TSP solutions.

Another reason for the above-mentioned neglect of cognitive complexity in the TSP literature may be related to the fact that linguistic representations of this particular problem have seldom been analyzed systematically so far. In other areas of wayfinding, linguistically sound analyses of verbalizations (going beyond a semantic content analysis) proved to be a particularly valuable way of accessing cognitive processes (Couclelis 1996; Denis et al. 1999). In this paper we combine features of the abstract TSP with selected complicating aspects (the presence of distractors and feature information), and we present a discourse-analytic investigation of verbal representations collected subsequent to the behavioral task, identifying how humans verbally summarize their experience and the cognitive processes involved in a coherent manner, and how these verbal data relate to the behavioral results collected.

Experiment 1

Experiment 1 was designed to test for the impact of two simple cognitive strategies that have been suggested to be involved in solving TSP-like planning tasks:

(i.) The Nearest Neighbor (NN) strategy, a simple yet efficient strategy for solving the TSPs: distances are minimized locally by always searching for the nearest neighbor which will then be visited next. Previous results suggest that humans do not simply follow this strategy but incorporate it into a more generalized procedure (Best and Simon 2000; Graham et al. 2000; Vickers et al. 2003). The present design addresses its relevance for humans' strategies in a scenario involving various colors and shapes.

(ii.) The cluster strategy, which states how path planning takes into account target clusters. In a first step, neighboring targets are conceptually combined to form clusters that are used to generate a coarse route plan. This plan is refined to account for navigation within a cluster during later planning steps. The cluster strategy furthermore predicts that navigators plan to visit as many targets as fast as possible, thus preferring to visit larger clusters first (Wiener et al. 2004). This hypothesis has not been tested in a TSP scenario before.

By the design of the spatial configurations, the behavioral data can shed light on the impact of each of these simple strategies. Note that both strategies are conceptually close in that both of them involve spatial proximity, though in different ways. The analysis of verbal data then allows for a more comprehensive investigation of the range of strategies involved in this kind of enriched TSP scenario, including a better understanding of how searching for the nearest neighbor relates to orienting towards object clusters. For instance, humans may either conceptualize object clusters, or focus predominantly on spatial directions guiding attention to those regions in which either many or few targets will be found. Thus, while the behavioral data highlight the impact of spatial proximity between objects (with respect to both cognitive strategies addressed), the verbal information sheds further light on the underlying reasons and conceptualizations.

Methods

Participants

Twelve students at the University of Bremen (age between 20 and 26 years, six of them female) agreed to participate in the experiment in exchange for course credit.

Materials

Each participant was given a square sheet of paper (21 × 21 cm) depicting a regular 5 × 5 grid of 25 symbols that differed in color and shape (see Fig. 1). During the experiment, participants had to solve 36 Traveling Salesperson tasks (TSPs). For each TSP they received a so-called 'shopping list' (see Fig. 1) depicting the symbol that defined the start place and the symbols that defined the target places that had to be visited for this particular TSP (ranging from 4 to 9 target places). Each target symbol was present in the grid exactly once. Each of the 36 different TSPs belonged to one of the following three types (see Fig. 2):

- **NN-adequate tasks**: For NN-adequate tasks the optimal solution is found by employing the nearest neighbor (NN) algorithm.
- **NN-inadequate tasks**: For NN-inadequate tasks, the NN algorithm does not predict the optimal but a clearly sub-optimal solution (see Fig. 2).

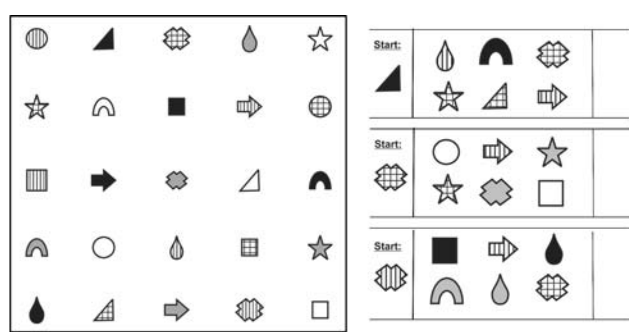

Fig. 1 The symbol field (*left*) and example shopping lists (*right*) for each of the three route types. Solutions see Fig. 2. The images are adapted to represent the original colours in *black*-and-*white* (*black*: *black* in the original; *white*: *yellow* in the original; vertical stripes: *red*: *grey*: *blue*; checked: *green*)

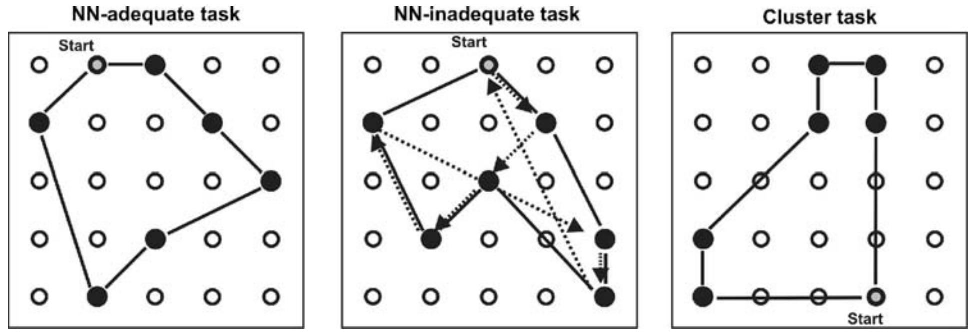

Fig. 2 Examples for the three route types. *Grey circles* represent starting points, *solid black circles* are target places, and the optimal paths are shown by *black lines*. The *dashed lines* in the NN-inadequate task depict the predictions of the NN strategy

- **Cluster (NN-ambiguous) tasks**: Here target places were distributed in two distinct target clusters of unequal size, situated at equal distance with respect to the start position. In contrast to NN-adequate and NN-inadequate tasks, these TSPs were NN-ambiguous: here the NN-algorithm did not predict a single but multiple solutions because of several situations in which the closest target places were equidistant from the current position.

The design was 6 (problem size) × 3 (task type), within-subjects, with two variants for each problem size and problem type. In other words, each participant solved 12 tasks of each type (NN-adequate, NN-inadeaquate, and cluster) in six different problem sizes, i.e., two different TSPs of each type involved four target places, two involved five, and so on, up to nine. Participants were not informed about the nature of the three different task types. To avoid systematic influences of the order in which the 36 TSP tasks were solved, the order was randomized for each participant. Also, to avoid an influence of a preferred trajectory direction (clockwise or counter-clockwise), the tasks were designed so as to balance the direction of the nearest neighbor and the largest cluster, respectively. Thus, any systematic influence of the task type cannot be ascribed to a preferred direction.

Predictions

If the NN strategy was involved in solving TSPs, we expected better performance in NN-adequate than in cluster tasks (NN-ambiguous) and in NN-inadequate tasks, measured by comparing the length of the solution paths in relation to the optimal (shortest) solution. If the cluster strategy was involved in solving TSPs, we expected that participants first visited targets in the richer (larger) cluster in those TSPs that involved clusters (cf. Fig. 2, right). Thus, the three different configurations addressed distinct aspects involved in attending to spatial proximity. Furthermore, due to the mental effort required to identify and remember the target positions in order to plan a suitable trajectory, we predicted better performance in smaller problem sizes than in larger ones, in spite of the fact that the overall number of targets was far lower than in most more abstract TSP tasks in the earlier literature.

Furthermore, we were interested in exploring how the participants' verbal accounts would reflect the behavioral results as well as further underlying procedures, given that little is known in the literature about the linguistic representation of cognitive strategies in TSP-related problems. Therefore the precise linguistic categories were developed post-hoc on the basis of the collected data (see below). Our general expectation was that if the behavioral data revealed an impact of the NN and the cluster strategies, we should find supporting evidence for these strategies in the linguistic representation. The NN strategy should be reflected by a substantial amount of expressions referring to spatial vicinity as well as by explicit reference to choices of the nearest symbol along the path. The cluster strategy should be expressed by utterances capturing several target symbols as a group, and by representations of the primacy of larger object clusters.

Procedure

The participants received the symbol grid together with a written instruction and 36 shopping lists in random order at the beginning of the experiment. An experimenter was present, ready to answer clarification questions. For each TSP, the participants' task was to find a route in the field that connects all symbols shown in the particular shopping list in the shortest possible way. The optimal order of connecting the symbols never corresponded to their order in the list. The instruction neither contained any mention of color nor of any particular strategy that could be used to simplify the task, but focused entirely on the explanation of materials and goals. Participants marked the symbols directly in the field with numbered little paper markers provided to them, thus receiving visual feedback concerning the symbols already visited. By these means it was ruled out that participants found individual ways of achieving such a marking (as they did in a pilot study we conducted beforehand). However, there were no connecting lines visually supporting the trajectory between the target symbols. The participants were asked to think carefully before producing the solutions but not to correct their decisions once they had placed the markers. They then noted the chosen order of symbols separately on a different sheet of paper.

After the participants solved this task for all of the 36 shopping lists, they produced linguistic representations of their solutions. In contrast to thinking aloud protocols collected during the task, this method ensures that the behavioral results were not influenced by the process of verbalizing strategies and procedures; furthermore, the participants were not distracted from solving the TSP tasks by having to produce verbal reports at intermediate stages. Also, the method is suitable for eliciting generalized procedures established gradually during task performance, abstracting from individual sub-tasks (Ericsson and Simon 1984; Tenbrink 2008). For present purposes such generalizations are valuable resources for identifying the range and complexity of strategies available to problem solvers. In particular, participants were asked to write two verbal accounts (see examples in the Appendix) according to the following instruction (translated here from German, the participants' native language in which the experiments were conducted):

> "I. Describe in as much detail as possible how you have solved the task, what you were considering while you solved it, and why you did it precisely in this way rather than another. Also, describe what, or which of the particular tasks, was easy and what was difficult for you during the task.
> II. Please write an instruction for a good friend of yours that allows him/her to solve the task as well as possible."

These instructions ensured that the participants did not simply copy the instructions given to them prior to solving the task, which did not include any hints as to how to best solve the task strategically. The participants could take as much time as they liked. The average time needed for the study was approximately 1 h 20 min.

Behavioral analysis

For each TSP, the length of the chosen path was calculated and compared to the optimal solution. Planning performance was described in percentage above optimal (PAO). A PAO value of 10 corresponds to a path that is 10% longer than the shortest possible path. Furthermore, the proportion of trials was assessed in which participants found the optimal (ie, the shortest possible) path.

10.6% of the solutions could not be analyzed because they did not contain the correct symbols given in the list: in some cases the numbered lists contained more symbols, in other cases fewer, or the symbol numbers were exchanged by mistake. The distribution across task types was approximately equal (cluster tasks: 12.5%; NN-adequate tasks: 10.4%; NN-inadequate tasks: 9.0%). This can be explained by the fact that symbol allocation errors could occur at each stage of the task, from path planning to the notation of the order of symbols. Participants did not receive feedback concerning the symbols covered in the solution; and the planning task involved a high amount of working memory load. Therefore, we did not include the amount of errors as a factor in our behavioral analysis, but focus on correctly (though not necessarily optimally) solved TSPs.

Linguistic analysis

The linguistic analysis combines both kinds of written accounts in this paper, since both of them were suitable to elicit strategies and processes that the participants recognized to be relevant for the task solution procedure. However, they differed with respect to the imagined audience, thus complementing each other: the second task encouraged the participants to establish ideas going beyond their actual achievements (see Tenbrink 2008 for an exemplary differential analysis of three elicitation types in a similar scenario). Thus, the data are suitable for an analysis of the range of variability available to (and verbalizable by) humans in this particular TSP setting, though they do not provide insight into the actual strategies that were employed in any single subtask.

The linguistic data were investigated using discourse-analytic methods. In particular, the analysis focuses on those aspects of language that systematically reflect the cognitive processes associated with strategies humans employ when solving TSPs. It combines insights drawn on the *content* level (analysis of conceptual strategies, representations, and processes directly described by the participants), with insights drawn on the *psychopragmatic* level (Dascal 1983), i.e., a pattern analysis of linguistic features of the verbal data. In other words, we look at *what* people say in relation to cognitive strategies along with systematic patterns in *how* they say it.

The verbalizations were broken down into informational units. These units are demarcated not syntactically but on the basis of task-related information (cf. Denis et al. 1999), such as reference to a particular consideration or action. An example is the following:

> (1) Dann habe ich mich nach dem Symbol gerichtet, das möglichst nah am Startsymbol lag, (2) um von dort aus gegen- oder mit dem Uhrzeigersinn den kürzesten Weg zu nehmen (3) und dabei alle Symbole "einzusammeln".

(1) then I focused on the symbol that was particularly close to the starting symbol, (2) in order to take the shortest path from there clockwise or anti-clockwise (3) and "collect" all of the symbols along the way.

Participants were free to produce as many units as they liked without any constraints with regard to their linguistic choices. The units served as basis for annotation according to a range of analytical categories relating to content as well as linguistic pattern analysis, as will be presented in detail below. Annotation was accomplished by two coders unaware of the hypotheses underlying this study. The inter-coder agreement score was >0.90 in a random sample of 13.5% of all data across analytical categories.

The quantitative distribution data resulting from the annotation procedure reflect the participants' relative attention (or lack of it) to a particular aspect. These effects are mediated by the processes' cognitive as well as lexical accessibility, i.e., the ease with which they can be verbalized, which is not directly accessible empirically. Therefore the main aim of the analysis is to establish the relationship of salient linguistic patterns to the behavioral results, as well as to earlier findings on cognitive strategies in the literature, to highlight systematic patterns in the ways in which the task solving procedure is represented in language, and to identify differences between experimental conditions (cf. comparative statistical tests reported in Experiment 2 below).

Behavioral results

Planning performance. Path planning performance was very good. On average, participants produced solutions that were only 2.35 percent above the optimal solution (PAO). Solutions were distributed, that is, for no TSP (except for two NN-adequate tasks with four targets) was there one single solution that was always used. An ANOVA [factors: number of targets (4–9) and type (NN-adequate, NN-inadequate, NN-ambiguous cluster] revealed significant effects for the number of targets ($F(5,57) = 5.57$, $P < 0.001$) as well as for the type of TSP task ($F(2,23) = 18.34$, $P < 0.001$), while the interaction did not reveal significant effects ($F(10,110) = 2.63$, $P = 0.07$) (see Fig. 3). Specifically, planning performance (PAO) was better for small (4, 5, 6 targets) TSPs than for large (7, 8, 9 targets) TSPs (post hoc test). Planning performance differed between types of TSP task (post hoc test): specifically, it was best for NN-adequate tasks, followed by NN-inadequate and by NN-ambiguous cluster tasks (see Fig. 3).

Found optimal path. A one way ANOVA revealed a significant main effect of the type of TSP task ($F(2,23) = 57{,}42$, $P < 0.001$). Specifically, performance of finding the optimal solution was best for NN-adequate tasks while it was comparable for NN-inadequate and NN-ambiguous cluster tasks (post hoc test).

Nearest neighbor strategy. The PAO predictions when using a NN-strategy were calculated for the different types of TSP tasks: for NN-adequate tasks it was obviously 0, for NN-inadequate tasks it was 16.92 and for NN-ambiguous cluster tasks it was 8.13 (averaged over multiple solutions since the NN-algorithm did not predict a single solution for NN-ambiguous tasks). Participants' PAO for both the NN-ambiguous tasks and the NN-inadequate tasks was significantly smaller than predicted by the NN-algorithm (NN-ambiguous: 2.55 PAO vs 8.13 PAO ($t(11) = 12.39$, $P < 0.001$); NN-inadequate tasks: 3.45 PAO vs 16.92 PAO ($t(11) = 27.58$, $P < 0.001$)). Thus, for both NN-inadequate and NN-ambiguous TSP tasks participants found shorter paths than the NN-algorithm predicts, showing that they did not entirely rely on a NN strategy.

Cluster (NN-ambiguous) tasks. In cluster tasks, the target places were distributed in two distinct clusters of unequal size. Overall, participants showed a small but significant preference to first visit the larger target cluster (58.18% vs. chance level [50%]: $t(11) = 2.99$, $P = 0.01$).

Linguistic results

Six of the twelve participants explicitly reported attending to spatial proximity, which is relevant for both the NN and the cluster strategy. The NN strategy is indicated, for

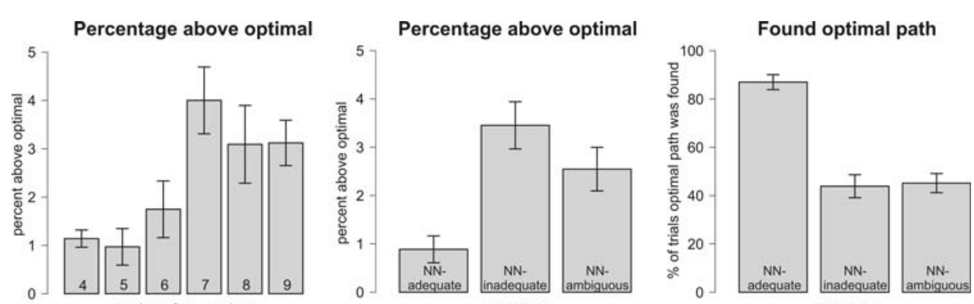

Fig. 3 Path planning performance, *left* as a function of the TSP size, *right* as a function of different types of navigation tasks

instance, by remarks that concern inspecting the symbols closest to the current position, as in: "Starting from this marking I looked at the surrounding symbols." In an early stage of planning this may involve looking for objects in close vicinity to each other, in other words, clusters of objects. An example for such a verbalization indicative both of awareness to spatial neighborhood (supportive of the NN strategy) and the cluster strategy (attending to larger groups of objects) is: "It was advantageous if several symbols were situated in groups. That is, directly beside or beneath each other." However, there are no indications in the linguistic data that participants consciously chose to visit the larger cluster first.

In order to gain insights concerning further aspects not directly targeted by the experimental design but relevant for the more general problem solving process, we categorized the participants' explicit references to strategies in relation to two major cognitive procedures (as already distinguished above), namely, *grouping and clustering* processes vs. *trajectory* features. Figure 4 gives the absolute number of mention of a strategy category by each participant. Next, we discuss a range of linguistic aspects related to particular strategies regardless if they are explicitly reported as a strategy. This discussion highlights what kinds of conceptualizations are associated with which of the processes (cluster-based vs. trajectory-based) in Fig. 4. For example, the category of cluster-based processes subsumes not only the NN- and cluster-related strategies just outlined (which were specifically targeted by the experimental design), but also conceptualizations of spatial regions and colour-based categorizations. The normalized frequencies of these various sub-processes as well as their distribution among individual participants are given in Table 2 (Appendix).

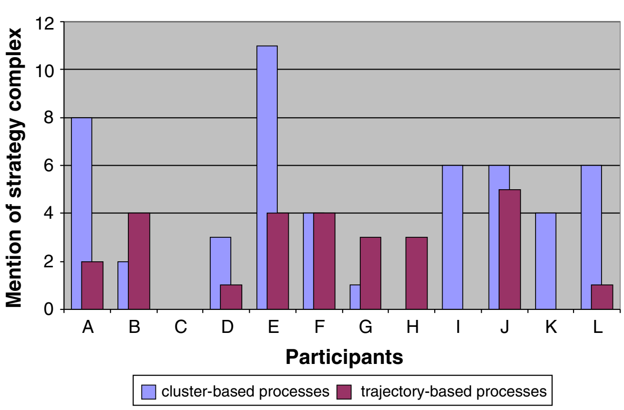

Fig. 4 Strategy distribution across participants: absolute number of times either cluster-based or trajectory-based processes were mentioned explicitly

Clustering and categorization processes

The linguistic data point to a cognitive differentiation between the processes of spatial clustering, regionalization, spatial neighborhood, and categorizations based on color or shape. Of these, color categorization turned out to be the strongest cognitive aspect. While proximity played a role for half of the participants as mentioned above, explicit reference to *groups* or *clusters* of targets was made by only two participants, while two other participants reported ordering the field into subareas such as *outer* vs. *inner* regions. Three participants reported paying particular attention to the target symbols' spatial relationship to the start (and end) position, which may represent a particularly prominent conceptual aspect as it needs to be visited twice (at the beginning and the end). Clearly all of these verbalizations, though related, describe different conceptualizations.

A different conceptual grouping strategy that is essentially non-spatial in nature is the mental *categorization* of targets. In the present design, the symbols could be categorized equally well according to either one of the features in which they differed (color and shape). Five of the twelve participants explicitly referred to a color-based procedure. Reference to the feature *color* appears in 11.27% of the informational units. In contrast, the linguistic item *shape* occurs only once at all.

Trajectory features

Related to earlier findings on *convex hull* or *trajectory-based* methods, the utterances of nine (of the 12) participants point to a gestalt-like conceptualization of the *trajectory* as a whole, either as a general process or with the specific association of (typically) a circle or (occasionally) a zigzag path. Related to this strategy is the somewhat simpler process of visualizing the position of the symbols in the field, which is mentioned by three participants. Also, two participants mention the aspect of *inserting residual* symbols into the overall trajectory. Attendance to the *crossing avoidance* strategy known from the literature was only reported by one participant.

A closer look at the linguistic features of the data reveals the ways in which the spatial path structure is represented, and how people conceptualize the process of obtaining a trajectory. Our participants referred to the path in various ways (cf. Table 3 in the Appendix). Of these, the mention of a "circle" or a "pattern" point most directly to a conceptual simplification process employed by the participants to build up a mental representation of the path. The references are linguistically associated to particular attributes as well as processes (summarized in Tables 4, 5, Appendix). The attributes represent the prominence of finding the *shortest path* and collecting *all* symbols along the way. The usage of verbs reflects action subprocesses: participants *search for* the trajectory of the path in the field; building up a path requires *planning* and *imagining* (where the former appears to be more prominent), and for some participants it involves a *choice* to be made. The *path building* process itself, which is the prominent result of the cognitive processes involved, involves *collecting* and *ticking off* the symbols on the list by combining them suitably. Thus, the overall path is a relevant factor throughout the whole process involving several subtasks.

Text structuring devices

Lexical markers such as *zuerst (first), wenn (if), dann (then), jedoch (however)* mark the functional relationships between clauses (e.g., Halliday 1994). In our context, they highlight how particular processes and ideas expressed in the clauses are related to each other. 20.9% of the informational units contain a temporal marker such as *first, then, now*, reflecting how a temporal structure is imposed on a complex and a priori unstructured task. Conditional enhancement markers such as *however* occur in 14.1% of the units; they account for the fact that one particular representation or strategy may not reflect the full story, or be valid across all possible situations. The fact that the participants frequently (in 13.3% of the units) refer to reasons (using markers such as *because*) highlights their goal-directed conscious decisions as well as their awareness of particular effects that need to be accounted for. Finally, their frequent use (22.1%) of extension and matter enhancement markers (such as *and, also, in addition*) shows the high variety of subtasks and processes involved in this particular endeavor. Here the absence of a temporally or logically subordinated structure suggests that the participants felt the need to account for a high number of processes and sub-tasks in parallel. The following statement illustrates some of these aspects:

> zuerst in der Reihenfolge auf dem Aufgabenzettel die Symbole auf dem Feld suchen und dabei bereits versuchen, sich "günstige" Verbindung bzw. beieinander gelegene Symbole zu merken [*first search for the symbols in the field according to their order on the list and in doing so already try to note a "convenient" connection or symbols that are located closely together*]

Here, the temporal marker *first* reflects a temporal structure, while the extension markers *and (in doing so)* and *bzw. ("or")* suggest the occurrence of several cognitive operations in parallel (searching for symbols along with the shortest path combining them as well as symbols located closely together).

Discussion

Overall, the participants' planning performance was remarkably good. While the choice of non-optimal routzes increased along with the number of targets to be visited, the average length of routes was only 3% above the optimal value even for the most complex TSPs. The TSPs were designed to test the impact of two planning strategies on performance in a complex planning task, the Nearest Neighbor (NN) strategy and the cluster strategy. Results provide support for both strategies. The participants found shorter paths when confronted with NN-adequate TSPs (for which the NN strategy actually yielded the optimal solution) as compared to NN-ambiguous and NN-inadequate TSPs. However, participants' performance for both NN-ambiguous and NN-inadequate tasks was better than predicted by the NN-algorithm. This result suggests that while the NN-strategy appeared to influence participants' behavior, it was not the sole strategy applied. Furthermore, in accordance with the cluster hypothesis, we found that if the target locations of a TSP could be assigned to two target clusters of unequal size, participants preferred to visit the larger target cluster first.

The relevance of spatial proximity (reflecting the strategies targeted here, NN and cluster, as cognitive subprocesses) was reflected linguistically by the fact that participants often referred to notions of spatial neighborhood. These references were tied into other conceptual aspects, supporting the hypothesis that the participants did not rely on spatial vicinity alone. Moreover, participants did not directly verbalize either the NN strategy or the cluster strategy. None of the references to spatial neighborhood reflected the unconditional heuristic of always choosing the nearest target symbol for the trajectory. Direct references to spatial groups of symbols were only infrequent, which can be explained by the fact that only a third of the tasks contained distinct object clusters; and participants did not report focusing on larger object clusters first, which mirrors the behavioral result that the size of the effect was fairly small (though significant). It can be concluded that the participants did attend to object clusters, but may not have been consciously aware of their preference for larger clusters. Also, since visiting the larger cluster first does not necessarily lead to shorter paths, this aspect may have been considered irrelevant. However, at least half of the participants utilized spatial vicinity as building blocks of their overall planning strategy, as expressed by the retrospective reports.

In spite of the enhanced complexity in our experimental design, these results generally match earlier findings based on more abstract TSP versions quite closely. To mention some examples, Graham et al. (2000) report a decline of performance with increased problem size, and Vickers et al. (2003) propose a model integrating nearest neighbor-related processes with a hierarchy of object clusters. Thus, in accord with our findings, humans may start out by focusing on spatial object clusters and only in a subsequent step plan their trajectory in detail (see also Basso et al. 2001). The *convex hull method* proposed by MacGregor et al. (2000) is, in effect, comparable to a clearly verbalized simplification strategy in our data, namely, conceptualizing trajectories which may be circle-like in shape and then inserting details. In our scenario, the procedure is more complicated in that it involves first identifying the target symbols in the field, which is not the case in a purely perceptually based process of pursuing a convex-hull trajectory. In contrast, the aspect of avoiding crossings did not seem to be particularly salient in the current setting, at least not as a conscious effort. While our specific cluster hypothesis is more closely related to findings outside the TSP area (cf. Wiener et al. 2004), the present findings in this regard do not seem to be in conflict with earlier TSP models. It is therefore likely that attending to spatial proximity as well as groups of objects will be relevant, though not the sole underlying cognitive processes, across diverse kinds of complex spatial planning tasks.

The participants' verbal reports further reflect the involvement of diverse strategies (including those not directly targeted by our design) in the problem solving process. Here, an interesting difference to earlier findings concerns the fact that, in our data, only one participant entirely failed to report any underlying strategy (cf. Fig. 4 above), in contrast to Vickers et al. (2001) where a substantial proportion of participants simply relied on perceptual processes, as the path seemed to be intuitively obvious. This supports the assumption that our more complex experimental design precludes the immediate perception of optimal trajectories and thus enhances the usage of cognitive strategies. Interestingly, the emerging strategies as such then do not seem to differ fundamentally from those identified for visually simpler TSP tasks. However, contrary to attempts in the literature to identify a global heuristic that is best suited to describe humans' strategies in solving the TSP, our participants apparently employed both trajectory-based and grouping strategies in flexible ways. In fact, most participants explicitly referred to various strategies without any apparent conflict (as might have been indicated, for instance, by linguistic markers of contrast or alternation; this was never the case). Accordingly, the distinct conceptions centering around object clusters and global trajectories are not competing but rather complementing aspects of problem solving.

The usage of temporal structuring devices allows for a generalized allocation of subprocesses to relative temporal positions. Participants start out by identifying, first, the position of the starting point, and then subsequently that of all other goal symbols within the field. Some people already simplify this process by focusing on candidates for the second and next-to-last positions in the trajectory by virtue of the symbols' spatial relationship to the start symbol. Identifying all symbols' positions in the field is followed by attempts to mentally visualize and memorize them; this process is often simplified, again, by focusing on a subset of symbols, this time by virtue of shared colors. At this point, participants attend to the fact that some symbols are situated close to each other, they notice object clusters, and some attend to particular subregions within the field. In such cases they tend to focus on the positions of the clusters and regions rather than single symbols. The next subprocess concerns the planning process proper, which involves mentally connecting the locations of the symbols. This process involves several kinds of simplifications, such as the visualization of a circle-like trajectory, connecting first all symbols of the same color, and focusing on those symbols that are particularly close to each other. These simplifications may be intertwined with each other. Remaining symbols are mentally inserted into the trajectory wherever it gets close. The final mental operation, before actually placing the markers of the trajectory, involves checking that the full set of symbols has been accounted for.

Of the reported subprocesses, those concerning searching, identifying, and memorizing positions in the field are not involved in scenarios in the literature such as abstract versions of the TSP which do not involve distractor objects. Also, in the absence of distinct features of the goals to be visited, strategies involving a focus on color are not available. Nevertheless, a substantial part of the problem in our design remains similar to the abstract TSP, as pointed out by Gärling et al. (1996) for naturalistic shopping scenarios.

In contrast to more simplistic approaches, the current intermediate scenario highlights the participants' creativity in breaking down the problem space into manageable units, utilizing particular features offered by the current scenario. For example, apart from purely *spatial* clustering processes, our participants also frequently focused on *color* as a shared object feature, which is not available in abstract scenarios. Obviously, the significance of the color feature (cf. Elliot and Maier 2007) overrides its irrelevance, as the distribution of colors in the field was entirely random. Moreover, the fact that our participants frequently referred to this strategy explicitly indicates that this does not only reflect an automatic perceptual attention effect (paralleling the results by Liqiang et al. 2004), but rather, a conscious decision to simplify the problem space by focusing on this salient feature. This result raises the question to what extent strategies triggered by an ubiquitous object feature such as color may influence other strategy choices in a complex spatial reasoning problem such as the TSP. For this reason, we conducted a second study in which the salience of color was purposely enhanced in order to enable a color-based spatial regionalization strategy. As in Experiment 1, we re-used a scenario introduced by Wiener et al. (2008b), which successfully addressed region-based strategies. We hypothesized that the participants' awareness of object clusters and spatial regions, both of which were shown to be marginally relevant in the first study, would be raised considerably by a color-based salience effect, yielding distinct behavioral as well as linguistic results. This study is reported next.

Experiment 2

Experiment 2 was designed to specifically test for the influence of region-based planning strategies when solving TSPs. In accordance with previous accounts of more abstract TSP problems, and with the verbal results gathered in Experiment 1, we hypothesized that path planning is not based on the detailed information about single locations only, but that environmental regions influence planning behavior. According to this view, target locations are first assigned to different regions in an environment; then, a coarse path is planned on the region level before it is refined concerning individual locations. The impact of this strategy is addressed via a salient regionalization of the environment, enabling a consistent use of the strategy which does not in all cases lead to optimal results. By choosing the intrinsically salient feature color as basis for regionalization, its role in human strategy choice is highlighted.

Methods

Participants

Twelve students at the University of Bremen (age between 19 and 23, nine of them female) agreed to participate in Experiment 2 in exchange for course credit. None of them participated in Experiment 1.

Materials

Materials were identical to Experiment 1 except that this time, symbols of equal color were neighboring each other in the grid, thus creating four clearly distinct regions (plus a single black square; see Fig. 5, left). The 36 TSPs (size between four and nine target symbols as before) belonged to the two following types (see Fig. 5, middle and right) that allowed for the investigation of region-based planning strategies (RS):

- **RS-adequate tasks**: The optimal solution for RS-adequate tasks involves first visiting all target places in one region before moving to the next region. Therefore, a region-based planning approach supports finding the optimal route in these cases, since only the shortest way of connecting targets inside a region needs to be considered.
- **RS-inadequate tasks**: Employing a region-based planning strategy on RS-inadequate tasks will systematically lead to sub-optimal paths, as the optimal solution requires leaving a region and re-entering it later. More specifically, if participants employed the proposed region-based strategy, it was expected that they produce more errors on the region-level (see below) when navigating RS-inadequate tasks as compared to RS-adequate tasks. Employing the region strategy on RS-inadequate tasks will systematically lead to fewer region crossings than the optimal solution.

The design was 6 (problem size) $\times$ 2 (task type), within-subjects, with three variants for each problem size and problem type. In other words, each participant solved 18 RS-adequate and 18 RS-inadequate tasks in six different problem sizes, i.e., three different TSPs of each type involved four target places, three involved 5, and so on. The order of the 36 tasks was random, and the participants were not informed about the nature of the two different task types.

Predictions

If participants employed region-based strategies, we expected better planning performance in RS-adequate than in RS-inadequate tasks. Furthermore, we expected systematic differences in the linguistic results as compared to Experiment 1; we expected the enhanced attentional focus on region-based planning procedures to be represented linguistically.

Procedure and analysis

The procedure was identical to Experiment 1. In addition to the analysis described above, participants' chosen paths were described and analyzed on the level of the regions. For example, the left path displayed in Fig. 5 can be described on the place level as follows: 20-18-17-21-16-7-13-9-10-20. On

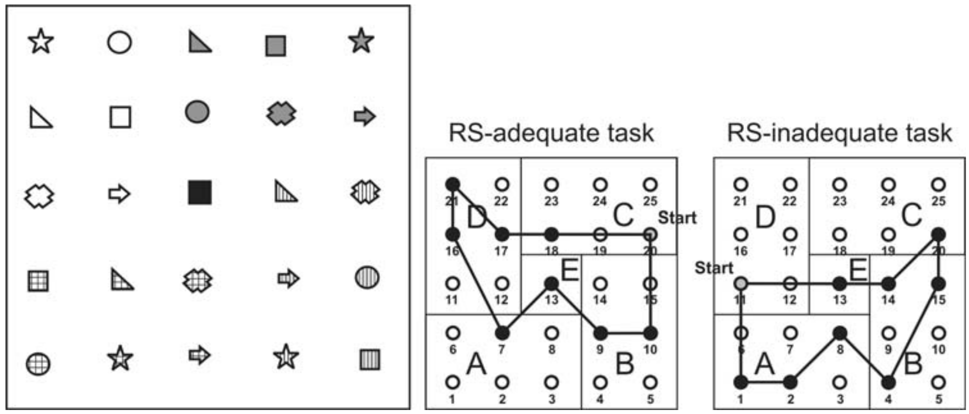

Fig. 5 Examples for the regionalized symbol field (*left*; the colours are represented as distinct *black*-and-*white* versions; equivalent to Fig. 1 above) and for the two route types (*middle and right*). Here *grey circles* represent starting points, *solid black circles* are target places, and the optimal solutions are shown by *black lines*. Navigating optimal solutions in RS-inadequate tasks requires leaving one region and re-entering it later. The two different route types were generated by mirroring and/or shifting the configuration of start and target places; otherwise the trajectories are identical

the region level the same path is represented as C–C-D-D-D-A-E-B–B-C. From this region representation, the number of region crossings was calculated for every chosen path as well as for all the corresponding optimal solution. Furthermore, by comparing the region representation of a chosen path with the region representation of the optimal solution, errors at the region level could be analyzed independent from errors at the place level. The linguistic analysis is additionally supported by comparative statistical tests. 7.6% of the TSPs could not be analyzed because they did not contain the correct symbols given in the list.

Behavioral results

On average, participants produced solutions that were 4.13% above the optimal solution (PAO). An ANOVA (factors: number of targets [4–9] and type [RS-adequate, RS-inadequate]) revealed significant main effects for the number of targets ($F(5,55) = 3.91$, $P < 0.01$) as well as for the type of TSP task ($F(1,11) = 6.27$, $P = 0.03$), while the interaction did not reveal significant effects ($F(5,56) = 1.20$, $P = 0.32$) (see Fig. 6). Specifically, planning performance (PAO) was better for small (4,5,6,7 targets) TSPs than for large (8,9 targets) TSPs (post hoc test). With respect to the type of TSP task, performance was better for RS-adequate tasks than for RS-inadequate tasks. Since there was no interaction, this effect was independent of the number of targets.

Furthermore, participants found the optimal path more often in RS-adequate than in RS-inadequate tasks ($t(11) = 5.05$, $P < 0.001$). Also on the region level, they found the optimal route more frequently on RS-adequate than on RS-inadequate tasks ($t(11) = 5.67$, $P < 0.001$). Furthermore, when solving RS-inadequate navigation tasks, participants crossed fewer region boundaries than required for optimal solutions. On average, they made 4.20 region transitions on RS-inadequate tasks, which is a reduction by 0.52 from the expected 4.72 region transitions for optimal solutions (4.20 vs 4.72, t-test: $t(11) = -11.13$, $P < 0.001$). On RS-adequate routes, they made 4.43 region transitions, which is an increase of 0.21 from the expected 4.22 region transitions for optimal solutions (4.43 vs 4.22, t-test: $t(11) = 5.99$, $P < 0.001$).

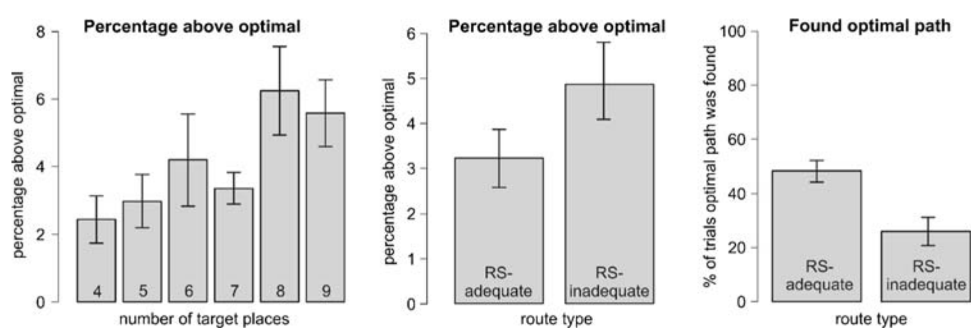

Fig. 6 Planning performance in relation to number of target places (*left*), route type (*middle*), and trials in which the optimal path was found

Linguistic results

Generally, the linguistic representations collected in this study resembled those of Experiment 1 in many respects, confirming the findings discussed above for a larger set of data. A two-sample t-test (collapsing across substrategies) yielded no significant difference between the relative frequencies of mention (cf. Fig. 4 above) of cluster-based processes in Experiment 1 as compared to Experiment 2 ($t(22) = -0.65$, $P = 0.52$), nor of trajectory-based processes ($t(21) = 0.72$, $P = 0.48$). Cluster-based processes were explicitly reported 4.25 times on average by each participant in Experiment 1 and 4.92 times in Experiment 2, while trajectory-based procedures were mentioned 2.25 times in Experiment 1 and 1.75 times in Experiment 2. In Experiment 2, the cluster-based strategy complex was significantly more prominent than the trajectory-based strategy complex (paired t-test: $t(11) = 3.21$, $P < 0.01$) as expected based on the prominence of object clusters in Experiment 2. In Experiment 1, the difference in relative frequencies between the two kinds of strategies did not reach significance (paired t-test: $t(11) = 1.67$, $P = 0.12$).

In the following we discuss the results that are pertinent for the particular features of the regionalized field, which should lead to an enhancement of the color categorization strategy. Results show that, on the one hand, the feature color and the associated color-based categorization strategy become much more prominent, and on the other hand, the color regions guide the application of other strategies as well. For instance, all evidence in the data pointing to a direction- or region-based spatial structuring process relies entirely on the color regions prominent in the field, as in the representative statement "Mostly I acted upon the color partitions." Additionally, in Experiment 1 all six (out of 12) participants who referred to a *spatial vicinity*-based strategy did so without linguistically entwining this idea with other strategies. In the present version, in contrast, this is only true for four of the nine participants (out of 12) who referred to this strategy. Four others related it directly to the color categorization strategy, as in: "If two colors are located beside each other, one needs to find neighboring symbols if possible", and the ninth (in referring to the only symbol that is not part of a color region) related it to the trajectory-based strategy: "I always included the black square into the 'circle' when it was closest to the circular line". In contrast to the non-regionalized version (Experiment 1) in which the strategy of focusing on color as a category was mentioned explicitly by only 5 of 12 participants, this time all participants except one reported attending to this factor. Altogether, reference to the feature *color* appears in 22.39% of the informational units in the data, reflecting its saliency, as opposed to only 11.27% in Experiment 1 (two-sample *t*-test: $t(22) = -2.12$, $P < 0.05$). In Experiment 1, there was no significant difference between reference to spatial proximity and reference to color (paired t-test: $t(11) = 1.05$, $P = 0.32$); in contrast, in the present condition the relevance of color was higher (paired t-test: $t(11) = 2.68$, $P < 0.05$).

Other strategies seem to be affected from this shift in focus as well. Relying on a particular trajectory as well as mentally visualizing the targets became less prominent (the former was mentioned by nine participants in Exp. 1 but only six in Exp. 2; the latter by three in Exp. 1 but only one in Exp. 2), while attention towards avoiding crossings (one in Exp. 1; six in Exp. 2) as well as attention to the relationship to the starting symbol (three in Exp. 1; six in Exp. 2) was enhanced. This latter strategy is closely related to the general problem of having to consider the return path. Seven of the 12 participants in Experiment 2 (but only two in Experiment 1) explicitly pointed out that one should not end up at the opposite side of the field. These remarks were formulated either as subtasks or as reasons for particular decisions that may seem to be counter-intuitive or even directly clash with other kinds of strategies (as one participant put it: "Note that 'closely situated' symbols sometimes should not be visited directly after one another if they are located on the return path anyway"). Conceivably, the prominence of the color categorization is seen as potentially distracting from the task of producing a closed trajectory.

Discussion

In this regionalized version of the TSP task, participants showed significantly better planning performance (PAO) in TSPs that were designed to support region-based planning strategies (RS-adequate tasks) than in TSPs for which region-based planning strategies predicted sub-optimal solutions. Moreover, for RS-inadequate tasks participants crossed fewer region boundaries than required for the optimal solution, and they made more errors on the region level than on RS-adequate tasks. This behavior was predicted if participants employed a region-based planning strategy. Taken together, the behavioral results support the hypothesis that, at least when confronted with salient regions in the environment, participants first plan their trajectory on a coarse level of granularity and then gradually refine it to include individual targets. This result is consistent with previous hierarchical accounts of abstract TSP planning strategies (e.g., Graham et al. 2000).

According to our results, participants attempt to reduce the complexity of a spatial problem by attending to salient object features, such as shared colors. While this is true even if this feature is irrelevant with respect to the spatial distribution (Experiment 1), it becomes considerably more prominent if the configuration supports the strategy. As a result, the strategies that relate to clustering processes become conceptually entwined with the color-based strategy, as reflected by the linguistic representations. Furthermore, the attention shift also affects the relative importance of some other conceptual aspects, such as trajectory visualizations and crossing avoidance. These effects demonstrate that the findings gained from the investigation of abstract problem solving tasks carried out at a computer screen, involving no feature-based differences, do not always generalize straightforwardly to more complex settings, although general cognitive procedures seem to be at least comparable.

Although participants' verbal reports clearly reflected an awareness that simply relying on the color regions does not always lead to optimal results, their behavioral performance was still better in the RS-adequate task, as expected. Obviously, a scenario supporting a particular simplification strategy leads to an overestimation of the degree to which this strategy may be useful.

General discussion

The present experiments focused on two main questions related to the investigation of traveling salesperson problems. One question concerned whether, or to what extent, cognitive strategies (or a particular subset of those) identified for abstract computer-based TSPs play a role in solving more complex and cognitively demanding problems that also involve combining a number of targets in an optimal trajectory before returning to the starting point. The other question concerned the linguistic reflection of cognitive strategies and the possibility of their combination, at least when represented in retrospective reports. In particular, we addressed the relative relevance of three strategies (*cluster, NN,* and *region-based*) in a TSP variant involving distractors as well as different colors and shapes, but no connecting lines. For this purpose we combined a paper-and-pencil version of the TSP which was specifically designed for testing the usage of these particular strategies with a linguistic analysis of retrospective reports.

The participants' behavioral results confirmed our hypotheses concerning, on the one hand, the importance of the region-based strategy as well as the cluster strategy, and on the other hand, the inadequacy of the Nearest Neighbor strategy to fully account for the participants' behavior. These results were reflected systematically in the linguistic representations. There was a high amount of reference to spatial vicinity, sometimes directly formulating the relevance of the nearest target objects; participants reported (in a somewhat less pronounced way) attending to object clusters, and their reports reflected a high degree of attention to the color regions in Experiment 2. Specifically, they regularly reported addressing colors in sequence, indicating a hierarchical planning procedure which was present to some degree also in Experiment 1 in which this strategy was not supported by the environment itself. The complexity of the verbal data furthermore reflected the fact that no single heuristic was sufficient to explain the behavioral results.

Generally, the participants' linguistic contributions revealed a very detailed and thorough account of the diversity of cognitive processes involved in the present TSP variant. Not only the strategies particularly addressed by our design but also most of the strategies known from the literature based on abstract TSP tasks were represented in one way or other in the linguistic data. Almost all of our participants referred to at least one kind of strategy that enabled them to simplify the problem complexity, either by focusing on object clusters or categories, or on a gestalt-like conceptualization of the overall spatial distribution, or both. These strategies are conceptually close to some of the algorithms proposed for computational TSPs, such as the *hierarchical nearest neighbor* (HNN) strategy (Vickers et al. 2003), the *convex hull method* proposed by MacGregor et al. (2000), and the *search algorithm* put forward by Best and Simon (2000). By the linguistic representations, we have gained a range of insights concerning how speakers verbally express cognitive processes of this kind, distributed throughout different steps of the incremental planning process (Hayes-Roth and Hayes-Roth 1979; Basso et al. 2001; Wiener et al. 2008a).

In a number of respects, the results gained from the linguistic analysis go beyond the findings that can be derived by behavioral data alone, since they reflect complex interrelationships not accessible to direct testing. Most prominently, TSP-related strategies have hitherto often been treated as real alternatives that are mutually exclusive. Our results indicate that they may be better represented as a repertory of strategies and subprocesses that are available to humans when solving TSP tasks. The relative weight of each particular subprocess or strategy may differ substantially between individuals and subtasks. A fair amount of combined strategies, and sometimes impressively complex conceptualization processes accounting for the limits of each particular one, emerged in the retrospective reports. The cluster strategy is intertwined with the nearest neighbor strategy, and the regionalization strategy can be merged with features of the targets to be visited, as shown by the results of the regionalized variant. Spatial neighborhood-based strategies are mediated by the need to return to the start position. While the particular complexity of the present scenario (as opposed to more abstract versions) certainly enhanced the emergence of multiple strategies, the verbal accounts are consistent with a general principle according to which humans utilize all kinds of simplifying procedures that they can think of when confronted with a complex problem solving task. In this regard, the proposal by Dry et al. (2006) to associate different kinds of strategies with either "bottom-up" perceptual processes or "top-down" cognitive processes may not reach far enough. Our participants employed simplifying conceptualizations with respect to each subtask involved in the overall procedure, from perceptual processes (such as identifying the symbols in the field), via spatial memory processes (temporarily remembering the position of the identified symbols), until the finalization of the planning process proper.

The specific impact of memory related processes on planning behavior and planning performance is the subject of another study (Wiener et al. 2008b), in which memory demands were systematically varied (no memory, working memory, working memory, and long term memory) in a navigational version of our current Experiment 2. Results show decreasing planning performance with increasing memory demands; at the same time, the reliance on region-based planning strategies increases, allowing for a reduction of both memory demands and planning effort.

A strategy-based TSP instruction

Inspired by the 'skeletal route description' proposed by Denis et al. (1999), we accumulated the cognitive processes reported by our participants, aiming at a cognitively adequate strategy-based linguistic instruction for solving this particular TSP variant. To achieve this, we extracted from the participants' contributions *subtasks and processes, strategies,* and *temporal allocations within a trial* for each variant if they were mentioned by at least two participants in that variant. Other contributions by the participants were neglected for the present purposes, such as reasons for particular decisions, remarks on different levels of difficulty and on effects of learning, and direct re-representations of the original task description. The resulting instructions are shown in Table 1.

Note that temporal allocations are tentative and nondeterministic, reflecting tendencies only; many subprocesses are not allocated a concrete temporal position at all. These can be interpreted as mental operators that are activated whenever relevant across subtasks.

Conclusions

In this study we investigated humans' verbal accounts of a range of strategies used in a TSP-like experimental task. The analysis combined behavioral trajectory analyses with discourse-analytic methods. Results reflect intricate

Table 1 Accumulated TSP instruction extracted from the participants' verbal contributions

Nonregionalized version (Exp. 1)	Regionalized version (Exp. 2)
First, identify the **starting point** in the field	First, identify the **starting point** in the field
Second, identify two of the target symbols that are relatively **close to the starting point** in the field. These can become the second and the next-to-last symbol in the trajectory, respectively	Second, identify two of the target symbols that are relatively **close to the starting point** in the field. These can become the second and the next-to-last symbol in the trajectory, respectively
Next, identify all other target symbols in the field, *and try to visualize their location in your head*. Focus on the **color** *and the form* of the symbols. Notice where the symbols are situated **closely together** in the field, *try to capture them as groups if possible, and focus on particular regions of the field*	Next, identify all other target symbols in the field. Focus on the **color** of the symbols. Notice where the symbols are situated **closely together** in the field, *particularly within one color region*
Then decide about the trajectory. Imagine combining the symbols in your head, forming a **circle**. Choose the **shortest route**, *combining symbols of the same* ***color*** *with each other*, and combining those symbols that are **close** to each other. Collect **residual** symbols along the way when your trajectory gets close	Then decide about the trajectory. Imagine combining the symbols in your head, forming a **circle**. Choose the **shortest route**, *going from one* ***color*** *to the next*, and combining those symbols that are **close** to each other. Collect **residual** symbols along the way when your trajectory gets close. *Avoid going back or crossing your own path*
Finally, **check** once again that you have considered all symbols in your trajectory	Finally, **check** once again that you have considered all symbols in your trajectory

Particularly salient key-words are marked in *bold*. The differences between the two descriptions according to experimental condition are highlighted in *italic* cognitive processes involving strategies for simplifying task complexity, in part resembling algorithms already identified in the literature for more abstract TSP versions. The present analysis reveals that, at least in the cognitively more demanding version targeted here, these strategies should not be regarded as competing theories accounting for humans' performance. Instead, they reflect complementary processes employed by humans to different degrees and potentially for different purposes or sub-tasks.

Future research should address, on the one hand, the question as to whether humans employ diverse strategies in flexible ways also in classical abstract TSP tasks, and on the other hand, the impact of further factors distinguishing computer-screen and paper-and-pencil versions from real-world navigation tasks, such as visual feedback via lines between targets. Thinking-aloud data collected during (rather than after) the task may contribute further insights about procedural and subtask-related aspects. For example, the relationship between particular configurations and prominent conceptual strategies requires further investigation. Based on the present linguistic analysis, which has been predominantly qualitative and explorative, targeted future research can provide more categorical and statistically further substantiated findings. Furthermore, an implementation of the strategies and processes identified here may lead to revealing performance comparisons with previous TSP algorithms. Finally, the possibility to train humans towards improved results using explicit strategy instructions should be explored.

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Appendix

Example texts by one participant (Exp. 2: regionalized version)

Text 1: (Describe in as much detail as possible how you have solved the task)

"Zuerst habe ich mir die Zeichen in einer beliebigen Reihenfolge auf dem Feld angeschaut und "geistig" miteinander verbunden, dann habe ich versucht herauszufinden, an welchen Stellen ich den Weg verkürzen könnte. Dabei war es meist sinnvoller, die Zeichen der gleichen Farbe nacheinander abzuhaken, weil das eigene Farbfeld meist näher liegt als ein Anderes. Oft habe ich festgestellt, dass die Zeichen, die ich als zweiten oder dritten Schritt gewählt hatte sich viel besser am Schluss eingliederten->darin sah ich auch eine der Hauptkomplikationen. Eine Andere ist das schwarze Quadrat, da es meist als Mitte fungierte und somit die Punkte größer über das Blatt verteilt waren".

First I looked at the symbols in a random order in the field and "mentally" connected them with each other, then I tried to find out at which places I could shorten the path. In doing this it was mostly more sensible to tick off the symbols of the same color one after another, because the same color region was mostly closer than another. Often I realized that the symbols that I chose as second or third step were much better suitable for the end->that's also where I saw one of the main complications. Another is the black square, since it typically served as midpoint and therefore the symbols were distributed more widely across the field.

Text 2: (Please write an instruction for a good friend of yours that allows him/her to solve the task as well as possible)

"Überlege dir, wie der kürzeste Weg über die auf der Liste aufgeführten Symbole aussehen könnte. Die Ausgangssymbol ist mit "Start" gekennzeichnet und ist gleichzeitig das Endsymbol. Achte vor allem auf das Ende der Strecke und überprüfe nochmals deine ersten Punkte. Wäre es vielleicht sinnvoller der Punkt am Ende einzugliedern? Denke nochmals nach, bevor du die Punkte legst. Wenn du einen Fehler gemacht hast, versuche ihn zu analysieren um ihn das nächste Mal zu vermeiden. Oft wird der kürzeste Weg erst offensichtlich, wenn die Punkte liegen, da du sie nicht verschieben darfst, versuche sie dir vorzustellen. Notiere anschließend die Zahlen vom zweiten Blatt (also die Positionen auf denen die Punkte liegen)."

Consider how the shortest path across the symbols shown in the list could look like. The starting symbol is marked as "start" and is simultaneously the end symbol. Pay particular attention to the end of the path and check again your first symbols. Could it be more sensible to integrate the symbol at the end? Reconsider again before you place the markers. If you have made a mistake, try to analyze it in order to avoid it next time. Often the shortest path only becomes obvious when the markers are placed, because you are not allowed to move them, try to imagine them. Note subsequently the numbers from the second sheet (that is, the positions on which the symbols lie).

Table 2 Strategy distribution in both experiments according to participants

Part. ID	word count	inf. units	number of strategies referred to	% Clustering	% Regionalization	% Color	% Shape	% Neighborhood	% Distance to start/end	**% cluster-based processes**	% Trajectory	% Insertion of details	% Crossing avoidance	% Visualization of targets	**% trajectory-based processes**
Exp. 1															
7	436	40	4	0.0	7.5	0.0	5.0	7.5	0.0	20.0	5.0	0.0	0.0	0.0	5.0
8	221	21	3	0.0	0.0	0.0	0.0	9.5	0.0	9.5	14.3	0.0	0.0	4.8	19.1
9	135	17	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
10	188	20	3	0.0	0.0	10.0	5.0	0.0	0.0	15.0	5.0	0.0	0.0	0.0	5.0
11	272	31	6	16.1	0.0	9.7	0.0	6.5	3.2	35.5	9.7	3.2	0.0	0.0	12.9
12	228	22	4	9.1	4.5	0.0	0.0	4.5	0.0	18.2	18.2	0.0	0.0	0.0	18.2
13	190	25	3	0.0	0.0	0.0	0.0	4.0	0.0	4.0	4.0	0.0	0.0	8.0	12.0
14	168	13	2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	15.4	0.0	0.0	7.7	23.1
15	185	15	2	0.0	0.0	33.3	6.7	0.0	0.0	40.0	0.0	0.0	0.0	0.0	0.0
16	275	27	5	0.0	0.0	14.8	0.0	0.0	7.4	22.2	7.4	7.4	3.7	0.0	18.5
17	135	14	2	0.0	0.0	0.0	0.0	14.3	14.3	28.6	0.0	0.0	0.0	0.0	0.00
18	178	18	3	0.0	11.1	22.2	0.0	0.0	0.0	33.3	5.6	0.0	0.0	0.0	5.6
Total	2611	263	37	2.7	2.3	6.8	1.5	4.2	1.9	19.4	7.2	1.1	0.4	1.5	10.3
Aver.	217.58	21.92	3.08												
Std. dev.	82.27	7.86	1.56							13.8					8.39
Mentioned by (no. of part.)				2	3	5	3	6	3		9	2	1	3	
Exp. 2 (reg.)															
19	231	21	5	0.0	0.0	28.6	0.0	4.8	9.5	42.9	4.8	0.0	9.5	0.0	14.3
20	401	47	5	0.0	0.0	6.4	0.0	4.3	2.1	12.8	4.3	0.0	2.1	0.0	6.4
21	212	18	3	0.0	0.0	5.6	0.0	0.0	0.0	5.6	5.6	0.0	0.0	5.6	11.1
22	219	22	3	0.0	0.0	0.0	0.0	18.2	9.1	27.3	0.0	0.0	4.5	0.0	4.6
23	160	18	5	0.0	0.0	11.1	0.0	5.6	5.6	22.2	5.6	0.0	11.1	0.0	16.7
24	216	18	5	16.7	0.0	38.9	0.0	5.6	0.0	61.1	0.0	5.6	5.6	0.0	11.1
25	278	29	5	0.0	0.0	10.3	0.0	3.4	0.0	13.8	10.3	3.4	3.4	0.0	17.2
26	311	28	4	3.6	0.0	14.3	0.0	7.1	3.6	28.6	0.0	0.0	0.0	0.0	0.0
27	206	18	3	0.0	0.0	5.6	0.0	5.6	0.0	11.1	11.1	0.0	0.0	0.0	11.1
28	209	21	3	0.0	0.0	9.5	0.0	9.5	4.8	23.8	0.0	0.0	0.0	0.0	0.0
29	100	11	1	0.0	0.0	9.1	0.0	0.0	0.0	9.1	0.0	0.0	0.0	0.0	0.0
30	145	13	1	0.0	0.0	15.4	0.0	0.0	0.0	15.4	0.0	0.0	0.0	0.0	0.0
Total	2688	264	43	1.5	0.0	12.1	0.0	5.7	3.0	22.3	3.8	0.8	3.0	0.4	8.0
Aver.	224	22	3.58												
Std. dev.	78.83	9.43	1.51							15.9					6.73
Mentioned by (no. of part.)				2	0	11	0	9	6		6	2	6	1	

Percentages are given in relation to informational units

Table 3 References to the path as trajectory: nouns (absolute numbers of occurrences)

Concept	Linguistic realizations	Exp. 1	Exp. 2 (reg.)
[Path, route]	Weg, Route, Linie	29	41
[Connection]	Verbindung	2	1
[Detour]	Umweg	2	2
[Circle, round tour]	Kreis, Rundgang	11	5
[Pattern]	Muster	6	1

Table 4 Features of the path: attributes (absolute numbers of occurrences)

Concept	Linguistic realizations	Exp. 1	Exp. 2 (reg.)
[Shortest possible]	kurz, kürz-, schnell(st)	16	24
[Various possibilities]	verschieden, divers, mögliche Wege	2	1
[No crossings]	schneid-, kreuz-	2	2
[Complete]	komplett, alle, ganz-	5	12

Table 5 Processes associated with the path: verbs (absolute numbers of occurrences)

Processes	Linguistic realizations	Exp. 1	Exp. 2 (reg.)
[Searching, finding]	suchen, finden	6	10
[Planning, thinking]	planen, austüfteln, überlegen, denken	11	7
[Imagining]	erahnen, aussehen, vorstellen	1	4
[Determining, choosing]	festlegen, auswählen, aussuchen	3	2
[Collecting, ticking off, combining]	einsammeln, abhaken, abklappern, verbinden, verknüpfen	13	10
[Travelling, producing]	ablaufen, gehen, nehmen, markieren, anordnen, bauen	12	16

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